

Indiana State Mathematics Contest
2015

Comprehensive

Do not open this test booklet until you have been advised to do so
by the test proctor.

This test was prepared by faculty at **Ball State University**

Next year's math contest date: Saturday, April 23, 2016

1. The parabola that passes through the points $(-1, 17)$, $(1, 1)$, and $(-2, 31)$ attains its minimum value when x equals:

- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

2. In the table below, how many rectangles of any size do not contain the symbol X ?

				X	
	X				

- (A) 22 (B) 86 (C) 92 (D) 106 (E) 125

3. How many positive integers have all of the following properties:

- I. less than 600 II. exactly 24 factors III. exactly 3 prime factors

- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

4. Begin with a Green Bag containing the numbers $\{1, 2, 3, \dots, 74\}$.
Take numbers from the Green Bag and place them one-at-a-time into a Red Bag.

What are the most numbers that you can place into the Red Bag such that no two numbers in the Red Bag will add to a multiple of 7?

- (A) 30 (B) 31 (C) 32 (D) 33 (E) 34

5. $\lim_{x \rightarrow \infty} \left(\frac{e^{-x}}{\ln(1 + 4e^{-x})} \right) =$

- (A) 0 (B) 0.13 (C) 0.23 (D) 0.25 (E) Does Not Exist

6. Given the following:

a, b, c, d are each 2 digit numbers

the 8 digits that make up a, b, c, d use the digits 1 through 8 exactly once each

$a < b < c < d$

$a + b + c + d = 144$

Find the minimum value of the product ad .

- (A) 408 (B) 624 (C) 636 (D) 728 (E) 848

7. Transportation to school for a rural county's 76 children is provided by a fleet of 4 buses. Drivers are chosen on a day-to-day basis and come from a pool of local farmers who have agreed to be "on call" when needed. Each driver has an 80% chance of being available if contacted. What is the smallest number of drivers that need to be in the pool if the county wants to have at least a 95% probability on any given day that all of the buses will run?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

8. Samuel Pepys asked Isaac Newton the following question: Which is most likely,

- I. to get at least one 6 when 6 dice are rolled
II. to get at least two 6s when 12 dice are rolled
III. to get at least three 6s when 18 dice are rolled

Answer Newton's question.

(A) I (B) II (C) III
(D) I and III are equal and the most likely (E) All are equally likely

9. My calculator has a Random Number Generator. I can give my calculator a lower bound and an upper bound, and it will generate a random number from within this range with the property that each number within this range has an equal probability of being generated. My calculator then displays as its output this number rounded to 2 decimal places. For example, typing $\text{RAND}(3, 5)$ into my calculator and pressing ENTER 3 times produces 4.24, 3.77, 5.00.

Suppose I type $\text{RAND}(-2, 8)$ into my calculator and press ENTER one time.

What is the probability that it will produce a number larger than 6?

(A) 0.1990 (B) 0.1995 (C) 0.2000 (D) 0.2005 (E) 0.2010

10. A, B, C, D are 4 workers who need to complete a task. If they complete the task alone, A can finish in 24 hours, B can finish in 20 hours, C can finish in 16 hours, and D can finish in 12 hours.

If they work in the order A, B, C, D, A, B, C, D, ... with each person working alone for 1 hour each turn, then how long it will take for them to complete the task?

(A) $15\frac{4}{5}$ hours (B) $17\frac{1}{6}$ hours (C) 18 hours (D) $18\frac{1}{6}$ hours (E) $20\frac{2}{3}$ hours

11. A, B, C are 3 different alcohol solutions whose alcohol concentrations are 48%, 62.5%, and $\frac{2}{3}$ respectively. The total volume of the 3 solutions is 100 liters. The volume of A is equal to the sum of the volumes of B and C. If you mix the 3 solutions, the alcohol concentration will be 56%. Find the original volume of alcohol in solution C.

(A) 12 liters (B) 15 liters (C) 18 liters (D) 21 liters (E) 24 liters

12. A bus runs from City A to City B. If the speed of the bus increases from its usual speed by one sixth, then it will get to City B twenty minutes faster than normal. If the bus runs at the usual speed for the first 72 km, but after that it increases its speed by one third, then it will get to City B 30 minutes faster than normal. Find the distance between City A and City B.

- (A) 504 km (B) 584 km (C) 612km (D) 827 km (E) 900 km

13. Let $f(x) = \frac{x}{\sqrt{1+x^2}}$ and define $f^{(2)}(x) = f(f(x))$, $f^{(3)}(x) = f(f(f(x)))$, and so on.

Then what is the value of $f^{(99)}(1)$?

- (A) $\frac{1}{10}$ (B) $\frac{1}{11}$ (C) $\frac{1}{\sqrt{83}}$ (D) $\frac{1}{\sqrt{99}}$ (E) $\frac{1}{\sqrt{9802}}$

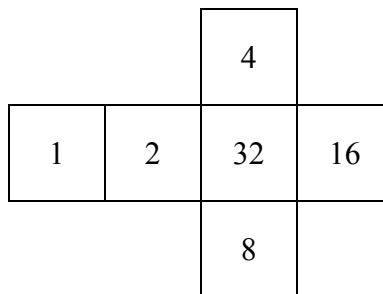
14. Urn 1 contains 3 White marbles and 4 Red marbles. Urn 2 contains 6 White marbles and 3 Red marbles. A biased coin, twice as likely to come up heads as tails, is flipped one time. If the coin shows heads, then 1 marble is taken from Urn 1. If the coin shows tails, then 1 marble is taken from Urn 2. You are told that a White marble was taken out of an Urn. What is the probability that the coin showed tails?

- (A) $\frac{7}{16}$ (B) $\frac{28}{37}$ (C) $\frac{1}{2}$ (D) $\frac{9}{37}$ (E) $\frac{1}{3}$

15. The cubic polynomial $x^3 - 3x + 52 = 0$ has roots α, β, γ . What is the value of $\alpha^2 + \beta^2 + \gamma^2$?

- (A) 0 (B) 3 (C) 6 (D) 29 (E) 52

16. Take the pattern below and form a cube. Then take three of these exact same cubes and stack them one on top of another on a table so that exactly 13 numbers are visible. What is the greatest possible sum of these 13 visible numbers?



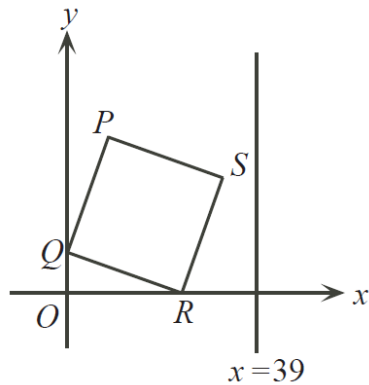
- (A) 154 (B) 159 (C) 164 (D) 167 (E) 189

17. Your teacher asks you to write down five integers such that the median is one more than the mean, and the unique mode is one greater than the median. You are then told the median is 10.

What is the smallest possible integer that you could include in your list?

- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

18. In the diagram below, square $PQRS$ has side length 25, Q is located at the point $(0, 7)$, and R is on the x -axis. The square is then rotated clockwise about R until S lies above the x -axis on the line with equation $x = 39$. The new coordinates of P will then be (a, b) . What is the value of $a + b$?



- (A) 46 (B) 50 (C) 54 (D) 62 (E) 71

19. In the grid below each row, column, and 3×3 box must contain exactly one of each digit from 1 to 9. What is the value of the center digit X?

3								7
	9		1		2			8
		8		6		5		
	7		6		1		5	
		2		X		7		
	3		5		9		4	
		3		8		4		
	8		7		4		2	
5								9

- (A) 2 (B) 3 (C) 4 (D) 8 (E) 9

20. On a strange farm there are dogs and cats living together. Given the following:

- I. there are 180 more dogs than cats
II. 20% of the dogs think they are cats
III. 20% of the cats think they are dogs
IV. 32% of all the dogs and cats think they are cats

How many dogs live on this strange farm?

- (A) 165 (B) 240 (C) 284 (D) 300 (E) 320

21. Spiders have 8 legs and no wings, Dragonflies have 6 legs and 2 pairs of wings, and Cicadas have 6 legs and 1 pair of wings. There are a total of 16 Spiders, Dragonflies, and Cicadas. All 16 have a total of 110 legs and 14 pairs of wings. What is the value of:

$$(\text{Number of Spiders}) \times (\text{Number of Dragonflies}) - (\text{Number of Cicadas})$$

- (A) 9 (B) 13 (C) 23 (D) 31 (E) 33

22. A sealed envelope contains a card with a single digit on it. Three of the following statements are true, and the other is false.

- I. The digit is 1
II. The digit is 2
III. The digit is not 3
IV. The digit is not 4

Which of the following must be correct?

- (A) I is false (B) II is true (C) II is false (D) III is false (E) IV is true

23. There exist positive integers A , B , C , with no common factors greater than 1, such that

$$A \log_{200}(5) + B \log_{200}(2) = C$$

What is the value of $2A + 3B - 4C$?

- (A) 8 (B) 9 (C) 10 (D) 11 (E) 12

24. Grass in a farmer's pasture grows at a constant and uniform rate. The grass can support 27 cows for 6 weeks or 23 cows for 9 weeks. How many weeks can the grass in this pasture support 21 cows?

- (A) 10 weeks (B) 11 weeks (C) 12 weeks (D) 13 weeks (E) 14 weeks

25. Which of the following integers can be expressed as the sum of 100 consecutive positive integers?

(A) 1,627,384,950

(B) 2,345,678,910

(C) 3,579,111,300

(D) 4,692,581,470

(E) 5,815,937,260

26. $\frac{2}{3!} + \frac{3}{4!} + \frac{4}{5!} + \dots + \frac{99}{100!} =$

(A) $\frac{1}{2} - \frac{1}{100!}$

(B) $\frac{1}{3} - \frac{1}{99!}$

(C) $\frac{1}{3} - \frac{1}{100!} + \frac{1}{99!}$

(D) $\frac{1}{2} - \frac{1}{100!} + \frac{1}{99!}$

(E) None of the above are correct

27. Let A be the set of the 2015 smallest positive multiples of 4, and let B be the set of the 2015 smallest positive multiples of 6. How many elements are in $A \cap B$?

(A) 169

(B) 336

(C) 503

(D) 671

(E) D 1004

28. If $\begin{bmatrix} 1 & 0 \\ 3 & -2 \end{bmatrix}^{23} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then the value of c is:

(A) -25,165,827

(B) -8,388,607

(C) -4,194,303

(D) 8,388,609

(E) 25,165,821