Indiana State Mathematics Contest 2015

Algebra I/Integrated Math I

<u>Do not open this test booklet until you have been advised to do so by the test proctor.</u>

This test was prepared by faculty at **University of Southern Indiana**

Next year's math contest date: Saturday, April 23, 2016

(1) Solve the equation for x: $2(x+3)-4(x-3)=-2$.

    (a) $\{-20\}$ (b) $\{-10\}$ (c) $\{10\}$ (d) $\{20\}$ (e) $\{40\}$

(2) Which of the following does <u>not</u> define a function?

    (a) $2x-3y=6$ (b) $\{(1,2),(2,3),(3,4)\}$ (c) $y=x^2$

    (d) $\{(1,2),(1,3),(1,4)\}$ (e) $y=\dfrac{1}{2x+7}$

(3) Determine the domain for the function: $f(x)=\dfrac{\sqrt{x+1}}{x-2}$.

    (a) $\{x|x \text{ is a real number}, x \neq 2, x \neq -1\}$ (b) $\{x|x \text{ is a real number}, x > -1\}$

    (c) $\{x|x \text{ is a real number}, x \neq 2, x \geq -1\}$ (d) $\{x|x \text{ is a real number}, x \geq -1\}$

    (e) $\{x|x \text{ is a real number}, x \neq 2\}$

(4) Suppose $f(x)=|x+4|-|x-5|$. Write $f(x)$ without using absolute-value notation if x is in $[-4,5)$.

    (a) $f(x)=2x-1$ (b) $f(x)=-2x-1$ (c) $f(x)=-2x+9$

    (d) $f(x)=-1$ (e) $f(x)=9$

(5) If $f(x)=-3x^2-7x+1$, find $2f\left(-\dfrac{1}{3}\right)$.

    (a) -2 (b) $\dfrac{8}{3}$ (c) $\dfrac{14}{3}$ (d) 6 (e) $\dfrac{22}{3}$

(6) Suppose that for some function f, $f(x+2)=-5x^2-8x+12$. Find $f(-1)$.

    (a) -57 (b) -9 (c) 15 (d) 25 (e) 81

(7) What is the ones digit for 3^{101} ?

    (a) 1 (b) 3 (c) 6 (d) 7 (e) 9

(8) Which of the following are true statements?

(i) The graph of a function $f(x)$ can have only one y-intercept.

(ii) The domain of the function $f(x) = \dfrac{\sqrt{3-x}}{x}$ does not contain 3 and 0.

(iii) The domain of the function $f(x) = \dfrac{\sqrt{3-x}}{x}$ does not contain 0.

(iv) Any vertical line must intersect the graph of a function exactly once.

(a) i and ii   (b) i and iii   (c) i and iv   (d) ii and iv   (e) i, iii and iv

(9) Which two functions have the same domain and the same range as each other?

(i) $f(x) = 5x - 2$ (ii) $f(x) = x^2 - 2$

(iii) $f(x) = |x| - 2$ (iv) $f(x) = \sqrt{x} - 2$

(a) i and ii   (b) i and iii   (c) i and iv   (d) ii and iii   (e) ii and iv

(10) The <u>sum</u> of the solutions to $|3x+1| = 2x+3$ is:

(a) -2 (b) $-1\dfrac{1}{5}$ (c) $1\dfrac{1}{5}$ (d) 2 (e) $3\dfrac{1}{4}$

(11) A line containing the point (a+8, 7) has an x-intercept of (19, 0). What is the value for 'a' if the equation of the line in slope-intercept form is $y = \dfrac{-7}{4}x + \dfrac{133}{4}$?

(a) 7   (b) $14\dfrac{1}{4}$ (c) 19   (d) 23   (e) None of these

(12) The entire real number line can be represented by which set of inequalities?

(a) $\dfrac{2x}{3} + 5 < 3$ or $6 - 3x < 15$ (b) $\dfrac{2x}{3} + 5 < 3$ or $6 - 3x \le 15$ (c) $\dfrac{2x}{3} + 5 \le 3$ and $6 - 3x < 15$

(d) $\dfrac{2x}{3} + 5 < 3$ or $6 - 3x \ge 15$ (e) $\dfrac{2x}{3} + 5 < 3$ and $6 - 3x \ge 15$

(13) Solve the equation for x: $3-6(2x-4)-4^2 = 4x-5(3-2x)$

(a) $\{-1\}$ (b) $\left\{\dfrac{-11}{13}\right\}$ (c) $\left\{\dfrac{23}{20}\right\}$ (d) $\left\{\dfrac{28}{13}\right\}$ (e) None of these

(14) What is the x-intercept of the line containing the points: (25, 33), (31, 29), and (40, 23)?

(a) $(47,0)$ (b) $\left(49\dfrac{2}{3},0\right)$ (c) $\left(70\dfrac{1}{2},0\right)$ (d) $\left(74\dfrac{1}{2},0\right)$ (e) None of these

(15) The graph of the inequality: $6\le|x+4|-y$ would include the following values along the x-axis:

(a) $\{x|-10\le x\le 2\}$ (b) $\{x|-2\le x\le 10\}$ (c) $\{x|x\le -2 \text{ or } x\ge 10\}$

(d) $\{x|x\le -10 \text{ and } x\ge 2\}$ (e) $\{x|x\le -10 \text{ or } x\ge 2\}$

(16) Chinese textile imports went up 1,000%. This makes the new import figure <u>N times</u> the original figure.

(a) N = 9   (b) N = 11   (c) N = 99   (d) N = 110   (e) N = 1,000

(17) If $y = 1.43x^2 - 3x + \sqrt{5}$, find the equation for the axis of symmetry.

(a) $x=-3$ (b) $x=-1.\overline{048951}$ (c) $x=\sqrt{5}$ (d) $x=\dfrac{150}{143}$ (e) None of these

(18) If the discriminant of a quadratic equation with rational coefficients is -15, the equation has:

(a) 1 real solution   (b) 2 imaginary solutions   (c) 2 irrational solutions

(d) 2 rational solutions   (e) No complex solutions

(19) Two solutions of the equation $Ax + By = 5$ are (-3, 2) and (4, -1). Find the sum of A and B.

(a) -14   (b) 0   (c) $\dfrac{15}{2}$ (d) 10   (e) None of these

(20) A basket of nuts contains only walnuts and pecans. Originally, the ratio of walnuts to pecans is 5 to 19. After 23 more pecans are added to the mix, the ratio of walnuts to pecans becomes 10 to 61. Which one of the following systems could be used to determine the original number of walnuts and pecans? Let w represent the original number of walnuts and p represent the original number of pecans.

(a) $\begin{cases} 10w = 61(p+23) \\ 5p = 19w \end{cases}$ (b) $\begin{cases} 61w = 10(p+23) \\ 5p = 19w \end{cases}$ (c) $\begin{cases} 61w = 10(p+23) \\ 19p = 5w \end{cases}$

(d) $\begin{cases} 10w = 61(p+23) \\ 19p = 5w \end{cases}$ (e) None of the these

(21) Which of the following systems is inconsistent?

(a) $\begin{cases} 2x + y = 4 \\ 4x + 2y = 8 \end{cases}$ (b) $\begin{cases} x + 2 = 6 \\ y = 5 \end{cases}$ (c) $\begin{cases} x - y = 10 \\ x + y = 1 \end{cases}$

(d) $\begin{cases} y = \dfrac{3}{4}x - 8 \\ y = \dfrac{4}{3}x + 1 \end{cases}$ (e) $\begin{cases} y - 5 = 0 \\ y = -12 \end{cases}$

(22) One serving of tomato soup contains 100 calories and 18 grams of carbohydrates. One serving of whole wheat bread contains 70 calories and 13 grams of carbohydrates. How many combined servings of soup and bread would be required to obtain 290 calories and 53 grams of carbohydrates?

(a) 3.5   (b) 15   (c) 55.85   (d) 70.4   (e) None of these

(23) Solve: $\begin{cases} \dfrac{1}{5}x + \dfrac{1}{2}y = 4 \\ \dfrac{3}{5}x - y = \dfrac{3}{4} \end{cases}$. Then find $x - y$.

(a) $\dfrac{137}{100}$ (b) $\dfrac{17}{4}$ (c) $\dfrac{35}{4}$ (d) $\dfrac{71}{4}$ (e) $\dfrac{181}{8}$

(24) Consider the system $\begin{cases} x+y=-2 \\ -3x+ky=-2k \end{cases}$. For what value of k will the system have

infinitely many solutions?

(a) -3　　　　(b) -1　　　　(c) 0　　　　(d) 3　　　　(e) None of these

(25) In a certain pet store, the cats sold for $10 each and the canaries for $15 each. The total value of the pets was $360. One night the owner forgot to close the door and the animals got loose. Two cats and half of the canaries disappeared. If the total value of the pets was now $220, how many cats were present before the escape?

(a) 10　　　　(b) 12　　　　(c) 16　　　　(d) 18　　　　(e) None of these

(26) You are standing in a line. There are twice as many people standing in front of you than are standing behind you. In the entire line, there are 7 more than the number of people standing in front of you. How many people are in the line?

(a) 6　　　　(b) 17　　　　(c) 18　　　　(d) 19　　　　(e) 22

(27) Which system of inequalities is graphed below?
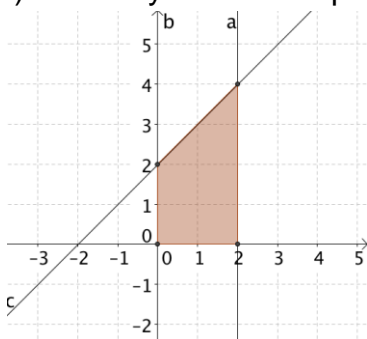
(a) $\begin{cases} x-y \geq -2 \\ 0 \leq x \leq 2 \\ y \leq 0 \end{cases}$ (b) $\begin{cases} x-y \geq -2 \\ 0 \leq y \leq 2 \\ x \geq 0 \end{cases}$ (c) $\begin{cases} x-y \leq -2 \\ 0 \leq x \leq 2 \\ y \geq 0 \end{cases}$ (d) $\begin{cases} x-y \leq 2 \\ 0 \leq x \leq 2 \\ y \geq 0 \end{cases}$ (e) $\begin{cases} x-y \geq -2 \\ 0 \leq x \leq 2 \\ y \geq 0 \end{cases}$

(28) At a party, everyone shook hands with everyone else exactly once. There were 66 handshakes. How many people came to the party?

(a) 12　　　　(b) 24　　　　(c) 33　　　　(d) 61　　　　(e) None of these

(29) Find the value: $3(4^{-1}+6^{-1})^{-1} - (2^{-1}+3^{-1})^{-1} =$

(a) $\dfrac{3}{7}$ (b) 1 (c) $1\dfrac{1}{7}$ (d) $2\dfrac{7}{60}$ (e) 6

(30) Find the distance between the points $(8,6)$ and $(-1,15)$.

(a) $\sqrt{35}$ (b) $2\sqrt{9}$ (c) $\sqrt{41}$ (d) $5\sqrt{5}$ (e) $9\sqrt{2}$

(31) The expression $(a^{5x}+b^{8y})^2$ is equivalent to which of the following:

(a) $a^{10x}+b^{16y}$ (b) $a^{25x^2}+b^{64y^2}$ (c) $a^{10x^2}+2a^{5x}b^{8y}+b^{16y^2}$

(d) $a^{10x}+2a^{5x}b^{8y}+b^{16y}$ (e) $a^{10x}+2(ab)^{13xy}+b^{16y}$

(32) If $r \geq 3$, then solve the following equation for r: $\sqrt{r^2-6r+9}=7$

(a) $r=3$ (b) $r=4$ (c) $r=6$ (d) $r=3+2\sqrt{5}$ (e) $r=10$

(33) Two cars are racing at a constant speed around a circular racetrack. Car A requires 15 seconds to travel once around the racetrack. Car B requires 25 seconds to travel once around the racetrack. If car A passes car B, how many seconds will elapse before car A once again passes car B?

(a) 10 Seconds   (b) 20 Seconds   (c) 32.25 Seconds

(d) 37.5 Seconds   (e) None of these

(34) Mona plans to invest $5,000 into two investments, part in a 5% simple interest account and the rest in a 6% simple interest investment. What is the most that she can invest at 5% and still be guaranteed at least $276.50 in interest after the first year?

(a) $2,050   (b) $2,250   (c) $2,350   (d) $2,650   (e) $3,250

(35) Correctly interpret the equation of the line that relates teen height and weight.

Height Vs. Weight

$y = 6.6102x - 276.84$
$R^2 = 0.9067$
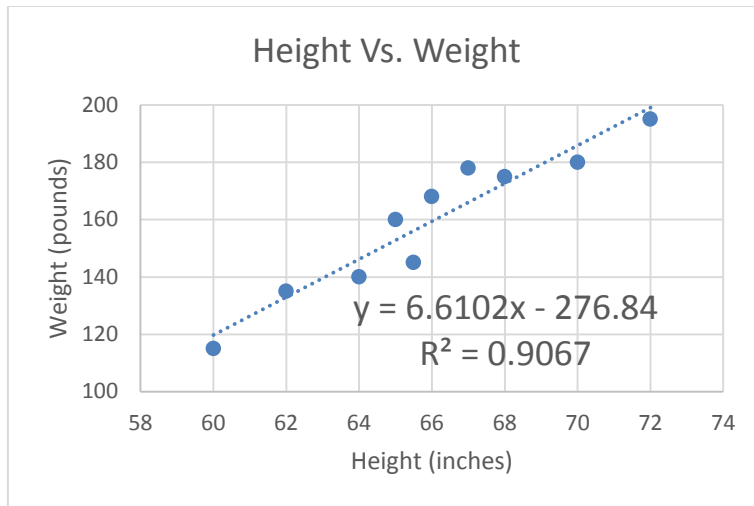
(a) If a teen grows an inch, they can expect to lose 276 pounds

(b) If a teen grows an inch, they can expect to gain 0.9067 lbs.

(c) If a teen grows two inches, they can expect to gain 6.6102 lbs.

(d) The y intercept of this line is 6.6102.

(e) If a teen grows an inch, they can expect to gain 6.6102 pounds.

(36) Jack and Jill share their food with Albert. Jack has 5 loaves of bread and Jill has 3 loaves. The three share the bread equally. Albert gives Jack and Jill 8 dollars, which they agree to share fairly. How much money should Jack receive?

(a) $1.00   (b) $2.00   (c) $3.00   (d) $4.00   (e) $7.00

(37) A quarter weighs the same as two pennies. If a pound of quarters is worth $25, then how much is a pound of pennies worth?

(a) $2.00   (b) $10.00   (c) $12.50   (d) $25.00   (e) $50.00

For the remainder of the test, assume all denominators to be non-zero

(38) Solve for p: $\dfrac{m-1}{p+3} = m+1$

(a) $p = \dfrac{2m-4}{m-1}$ (b) $p = \dfrac{-2m-4}{m+1}$ (c) $p = \dfrac{m+1}{-2m-4}$

(d) $p = \dfrac{-2m+2}{m+1}$ (e) $p = \dfrac{m+1}{2m-4}$

(39) Find the solutions for the equation: $bx^2 - cx = d$.

(a) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (b) $x = \dfrac{-c \pm \sqrt{c^2 - 4bd}}{2b}$ (c) $x = \dfrac{c \pm \sqrt{c^2 + 4bd}}{2b}$

(d) $x = \dfrac{c \pm \sqrt{-c^2 - 4bd}}{2b}$ (e) $x = \dfrac{-d \pm \sqrt{d^2 - 4bd}}{2c}$

(40) When dividing the polynomials: $\dfrac{2y^3 - 3y^2 - 8}{y+4}$, the remainder will be:

(a) -96   (b) -80   (c) -8   (d) 44   (e) None of these