Indiana State Mathematics Contest 2015

Algebra II/Integrated Math II

Do not open this test booklet until you have been advised to do so by the test proctor.

This test was prepared by faculty at **Butler University**

Next year's math contest date: Saturday, April 23, 2016

1) Multiply and simplify: $\frac{3x^2+5x-2}{x^2-4} \cdot \frac{3x^2-12}{12x-4}$

   A) $\frac{3x+6}{4}$ B) $\frac{3}{4}$ C) $\frac{(5x-2)(x-3)}{16}$

   D) $\frac{9x^4+15x^3-44x^2-66x+24}{4(x+2)(x-2)(x-1)}$ E) none of these

2) Solve for x and y and express your answer as the sum of x and y.

   $$\frac{12}{x} - \frac{12}{y} = 7 \qquad \frac{3}{x} + \frac{4}{y} = 0$$

   A) 1   B) 2   C) -2 D) -1 E) -12

3) Find the indicated infinite sum. $\sum_{n=0}^{\infty} 4\left(\frac{1}{4^n}\right)$

   A) $\frac{16}{3}$ B) 3   C) 1   D) $\frac{62}{3}$ E) 4

4) Given $f(x) = \frac{x^2-x}{x+1}$, find $f(1+i)$.

   A) $\frac{1+3i}{5}$ B) $\frac{-1+3i}{3}$ C) $\frac{-1+3i}{5}$ D) $-1+3i$ E) none of these

5) Find the interval(s) on the Real number line for which the radicand is greater than 0 for $\sqrt[4]{81-3x^2}$.

   A) $(3, 3)$ B) $[3, 3]$ C) $[-3\sqrt{3}, 3\sqrt{3}]$ D) $x = \pm 3$ E) $\left(-3\sqrt{3}, 3\sqrt{3}\right)$

6) For $\frac{x^2}{36} + \frac{y^2}{64} = 1$, the equation of an ellipse, find one of the foci.

   A) $\left(0, 3\sqrt{10}\right)$ B) $(0, 4)$ C) $\left(0, 2\sqrt{7}\right)$ D) $(6, 0)$ E) None of these

7) What is the y-intercept of the line that passes through the point $(1, 0)$ and is perpendicular to the line with the equation $x + 2y - 3 = 0$.

A) $(0, -1)$ B) $(0, 0)$ C) $(0, -2)$ D) $\left(0, -\frac{3}{2}\right)$ E) $(2, 0)$

8) The sum of two numbers is 25. The sum of their squares is 313. What is the difference between the two solutions?

A) 5   B) 4   C) 3   D) 2   E) 1

9) If $x + y = z$ and $x = y$, then all of the following are true except:

I. $2x + 2y = 2z$ IV. $x = \frac{z}{2}$

II. $x - y = 0$ V. $z - y = 2x$

III. $x - z = y - z$

A) I   B) II   C) III   D) IV   E) V

10) Solve the inequality $\frac{x+2}{x-1} \leq 2.$

A) $(-\infty, 1) \cup [4, \infty)$ B) $(-\infty, 1] \cup (4, \infty)$ C) $(-\infty, 1) \cup (4, \infty)$

D) all Reals except x = 1   E) none of these

11) Evaluate $\sqrt{-36}\left(\sqrt{-9} + \frac{\sqrt{12}}{\sqrt{-27}}\right).$

A) 14   B) $\frac{52}{3}$ C) -22 D) -14 E) none of these

12) Solve $8^{4t-2} = 32^{t+1}.$

A)   6   B) $\frac{11}{7}$ C)   0   D) 4.144   E) no solution

13) Find the domain of the function $f(x) = \frac{x-2}{x+5} - \frac{4}{\sqrt{3-x}}$.

   A) $x > 3$, $x \neq 5$ B) All reals, $x \neq 3, 5$ C) $x < 3$

   D) All reals, $x \neq 5$ E) $x < 3, x \neq -5$

14) Simplify $\left(\frac{x^{-1/2}y^{3/4}}{y^{-5/4}}\right)^4$.

   A) $x^2 y^8$ B) $\frac{y^8}{x^2}$ C) $\frac{1}{x^2 y^2}$ D) $\frac{x^{-2}}{y^{-8}}$ E) none of these

15) If $a * b = \frac{a-b}{a+b}$, find $2 * (3 * 1)$.

   A) $\frac{3}{5}$ B) $\frac{1}{3}$ C) $-\frac{3}{2}$ D) 1   E) 6

16) Identify the numerator of the repeated decimal 1.185185… when written as a rational number in the form $\frac{a}{b}$.

   A) 999   B) 1184   C) 1000   D) 1851   E) 1185

17) A combination lock will open when the right order of 3 numbers (from 1-40 inclusive) is selected. How many different settings are possible?

   A) 59,280   B) 9880   C) 64,000   D) 120   E) 117

18) If $f(x) = \frac{5}{2x} + 7$, find $f^{-1}(0)$ if possible.

   A) 4   B) -5/14   C) 33/5   D) $\frac{23}{2}$ E) not possible

19) Solve $x + 2\sqrt{x-1} = 0$.

   A) 2   B) 1   C) 1, 2   D) 0   E) no solution

20) Given the graph of the function $f(x)$, what would the graph of $f(x + 2) - 3$ look like?

    A) same shape, moved down 2 and to the left 3

    B) same shape, moved down 3 and to the left 2

    C) same shape, moved up 2 and to the right 3

    D) same shape, moved up 3 and to the right 2

    E) same shape, moved down 3 and to the right 2

21) At noon Tom left his house and drove at a constant speed of 25mph to his friend Ann's house which is 38 miles away. Twelve minutes after Tom left his house, Ann left her house and drove at a constant speed of 35mph towards Tom's house. What time would they meet?

    A) 12:15   B) 12:20   C) 12:35   D) 12:45   E) 1:00

22) Find the roots of $x^2 + 4x + 13 = 0$.

    A) $-2 \pm 3i$ B) 1, -5   C) $-2 \pm 6i$ D) -1, 5   E) none of these

23) The total resistance, R, of two resistors, A and B, connected in parallel is given by the formula

$\frac{1}{R} = \frac{1}{A} + \frac{1}{B}$. Solve for B.

    A) $\frac{1}{R} - A$ B) $\frac{RA}{R-A}$ C) $R - A$ D) $\frac{RA}{A-R}$ E) none of these

24) The mean of a set of 10 numbers is 40. One number was incorrectly recorded as 20 instead of 2. What should be the corrected mean?

    A) 18   B) 38.2   C) 38   D) 22.8   E) none of these

25) If $f(x) = x^2 - 2$ and $g(x) = 2x + 2$, find $f\left(g\left(f\left(\frac{1}{2}\right)\right)\right)$.

    A) $\frac{1}{2}$ B) $\frac{1}{4}$ C) 1   D) 2   E) 4

26) For which numbers is a number minus its square positive?

   A) $(0,1)$ B) $(-\infty,0) \cup (1,\infty)$ C) $(0,\infty)$ D) $(-\infty,0] \cup [1,\infty)$ E) $(-\infty,0)$

27) Find the remainder when $x^3 - x^2 - 14x + 1$ is divided by $x - 4$.

   A) 9   B) 1   C) -23 D) -7 E) none of these

28) Find $\log_4 \left(\frac{c}{a}\right) - \log_4 \left(\frac{b}{a}\right) + \log_4 \left(\frac{b}{c}\right)$.

   A) 1   B) 0   C) 4   D) $\dfrac{c^2 - bc + ab}{ac}$ E) undefined

29) Find the center and radius of the circle $x^2 - 6x + y^2 + 8y = 11$.

   A) $(-6,8)$ and $\sqrt{11}$ B) $(-3,4)$ and 6   C) $(3,-4)$ and 36

   D) $(3,-4)$ and 6   E) none of these

30) If the number written in base 3 is 210100, how is the preceding number written?

   A) 110100   B) 210000   C) 210021   D) 210099   E) 210022

31) $(x - 2y + z)^2 =$

   A) $x^2 - 4y^2 + z^2$

   B) $x^2 + 4y^2 + z^2$

   C) $x^2 + 4y^2 + z^2 - 4xy + 2xz - 4yz$

   D) $x^2 + 4y^2 + z^2 + 4xy - 2xz - 4yz$

   E) none of these

32) Rationalize the denominator of $\frac{3-\sqrt{x}}{4+\sqrt{x}}$.

A) $\frac{12-7\sqrt{x}+x}{16-x}$ B) $\frac{12+x}{16-x}$ C) $\frac{3-x}{4+x}$ D) $\frac{12-\sqrt{x}+x}{4-x}$ E) $\frac{9-x}{12+7\sqrt{x}+x}$

33) Assume dog-birds have 2 feet and 2 heads, but cat-birds have 4 feet and 1 head. Looking into a cage of dog-birds and cat-birds, you count 36 feet and 24 heads. How many birds are in the cage?

A) 12   B) 13   C) 14   D) 15   E) 16

34) Suppose the surface area of a sphere equals twice the numerical value of its volume. What is the sphere's radius?

A) 0.75   B) 2   C) 1.5   D) 3   E) 1

35) Solve $\ln x^2 = (\ln x)^2$.

A) 1   B) $1, e^2$ C) $0, e^2$ D) e^2 E) no solution

36) If 80 mg of a radioactive substance decays to 5 mg in 8 minutes, find the half-life in minutes of the substance.

A) 8   B) 4   C) 1   D) 3   E) 2