Indiana State Mathematics Contest 2016

Algebra II/Integrated Math II

Do not open this test booklet until you have been advised to do so by the test proctor.

This test was prepared by faculty at **Franklin College**

**Next year's math contest date: Saturday, April 22, 2017**

Algebra II

1. What is the reciprocal of $4 + 3i$?

   a. $\frac{4}{25} - \frac{3}{25}i$ b. $\frac{4}{25} + \frac{3}{25}i$ c. $\frac{1}{4} - \frac{1}{3}i$ d. $\frac{1}{4} + \frac{1}{3}i$ e. $\frac{3-4i}{7}$

2. In the song "The Twelve Days of Christmas," the true love gives 1 gift on the first day, 3 gifts on the second (2 turtle doves and a partridge in a pear tree; 2+1=3), and so on. In total, how many gifts are given for all 12 days?

   a. 468 b. 364 c. 256 d. 182 e. 78

3. If (a, b) and (c, d) are the points of intersection between the hyperbola $x^2 - y^2 = 4$ and the line $2y - x = 2$, find the value of $a + b + c + d$.

   a. $-2\frac{2}{3}$ b. 4 c. $5\frac{1}{3}$ d. 7 e. 8

4. Rewrite the expression $3\log_8(x + 2) - \log_8(x)$ as a single expression. (Assume $x > 0$.)

   a. $\log_8\left(\frac{3(x+2)}{x}\right)$

   b. $\log_8\left(\frac{(x+2)^3}{x}\right)$

   c. $\log_8(2x + 2)$

   d. $\log_8(2x + 6)$

   e. $\log_8\left(\frac{x+2}{x}\right)^3$

5. At Spy High, students are enrolled in either exactly two of the school's three after school clubs or zero after school clubs. $\frac{1}{2}$ of the students are in the Codebreaking Club, $\frac{1}{2}$ of the students are in the Gadget Design club, and $\frac{1}{2}$ of the students are in Spies' Disguise Club. If you were to pick a student at random from the school, what is the probability that they are not involved in any clubs?

   a. $\frac{1}{2}$ b. $\frac{3}{8}$ c. $\frac{1}{3}$ d. $\frac{1}{4}$ e. 0

6. Solve for x, if $|x - 5| < 1$.

   a. $x < 4$ or $x > 6$

   b. $4 < x < 6$

   c. $-6 < x < -4$

   d. $x < -6$ or $x > -4$

   e. $-6 < x < 6$

7. Which of the following are functions?

I.

Player	Points
Wilt	100
Kobe	81
Wilt	78
David	73
Wilt	73

II.
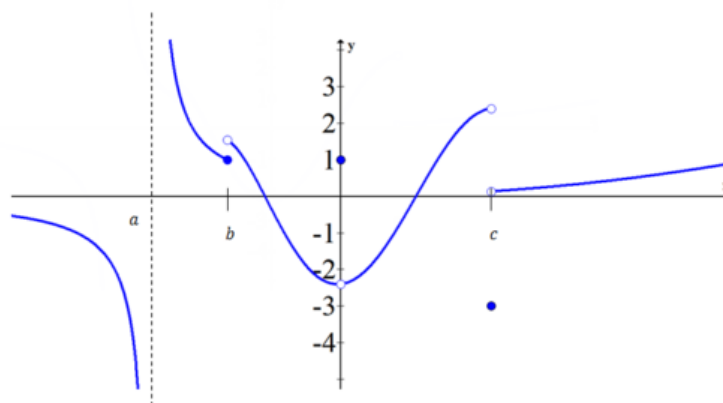
III.

$$y = \pm\sqrt{1 - x^2}$$

   a. Only II   b. I and II   c. I and III   d. II and III   e. None

8. Adam Vinatieri lines up to attempt a 60 yard field goal aimed at the goalpost, whose crossbar is 10 feet above the ground. If he kicks the ball directly at the goalpost with a horizontal velocity of 90 ft/s, and if the height of the ball in feet at t seconds is given by $h(t) = -16t^2 + 40t$, determine the result of the field goal kick.
    a. It will land about 15 feet short of the goal post.
    b. It will land about 6 feet short of the goal post.
    c. It will go above the crossbar by about 16 feet
    d. It will go above the crossbar by about 6 feet.
    e. It will go under the crossbar by about 4 feet.

9. If (a, b, c) is a solution to $x + 2y + 3z = -1$, $2x + 3y + z = -4$, and $3x + 2y + z = 1$, find $a + b + c =$.
    a. -8 b. -2 c. 0   d. 5   e. 6

10. For a certain equation, z varies jointly with y^2 and inversely with x. When $x = 8$ and $y = 2$, then $z = 6$. What will x equal if $y = 3$ and $z = 18$?
    a. 2   b. 3   c. 4   d. 5   e. 6

11. If a and b are the two unique solutions of $8^{x+2} \times 4^{4-x^2} = 1$, find $a \times b$.
    a. -14 b. $\frac{4}{7}$ c. 2   d. -7 e. No solution

12. Find the equation of the graph obtained by reflecting $y = \frac{x+2}{4-x}$ over the line $y = x$.
    a. $y = \frac{4x-2}{x+1}$ b. $y = \frac{2+x}{x-4}$ c. $y = \frac{4x+2}{x-1}$ d. $y = \frac{4-x}{x+2}$ e. Cannot be determined

13. To obtain the graph of the function $g(x) = 2^{x+3} - 4$ from $f(x) = 2^x$, how would you need to shift the graph of $f(x)$?
    a. Down 4 and left 3
    b. Down 3 and right 4
    c. Up 4 and right 3
    d. Down 4 and right 3
    e. Up 3 and left 4

14. Solve for x if $\sqrt{3x - 2} = \sqrt{x} + 8$.
    a. 9   b. 9, 121   c. 121   d. 9, 64   e. No Solution

15. For which value of x is $\sqrt[4]{x^3}$ a rational number?
    a. $\frac{81}{8}$ b. $\frac{32}{125}$ c. $\frac{16}{81}$ d. $-\frac{1000}{8}$ e. i

16. Solve for x if $\frac{1}{x+4} + \frac{1}{x-4} = \frac{16}{x^2-16}$.
    a. 2   b. -4   c. 4   d. 8   e. No Solution

17. If $f(x) = x^2 - 17$ and $g(x) = \sqrt{x} - 5$, find $g(f(-9))$.
    a. -13 b. $\sqrt{59}$ c. 3   d. 9   e. Does not exist

18. How many zeros does the following piecewise function have?
$$f(x) = \begin{cases} 4x + 2 & if\ x \le 0 \\ x^3 - 5x^2 + 6x & if\ 0 < x < 4 \\ 2x^2 + 7x - 72 & if\ x \ge 4 \end{cases}$$

    a. 6   b. 4   c. 3   d. 5   e. 2

19. For the equation $f(x) = mx + b$, for which values of m and b does $f(x) \ne f^{-1}(x)$?
    a. $m = 1$, $b = 0$
    b. $m = -1$, $b = 0$
    c. $m = 1$, $b = 5$
    d. $m = -1$, $b = 5$
    e. $m = -1$, $b = \pi$

20. How many real solutions are there to $|x^2 - 9| = |3x + 9|$?
    a. 3   b. 1   c. 4   d. 2   e. 0

21. Write the related exponential equation for $\log_2 x = 5$. (Assume $x > 0$.)
    a. $5^2 = x$ b. $5^x = 2$ c. $2^5 = x$ d. $2^x = 5$ e. $x^5 = 2$

22. Find the terms of the sequence defined by $a_n = n^2$, where $1 \leq n \leq 4$.

    a. $1, 2, 3, 4$ b. $0, 1, 4, 9$ c. $1, 3, 5, 7$ d. $4, 9, 16, 25$ e. $1, 4, 9, 16$

23. $(x - y + z)(x - y - z) =$

    a. $x^2 + y^2 - z^2 - 2xz$
    b. $x^2 + y^2 - z^2 - 2xy$
    c. $x^2 + y^2 - z^2 - 2yz$
    d. $x^2 + y^2 - z^2 + 2xz$
    e. $x^2 + y^2 - z^2 - 2xz - 2yz$

24. Simplify $\left(\frac{xz^0}{3y^3}\right)^{-2}$.

    a. $-\frac{2x}{3y^3}$ b. 0 c. $\frac{x^{-1}z^{-2}}{6y}$ d. $\frac{9y^6}{x^2}$ e. $-\frac{x^2}{9y^6}$

25. Which of the following functions does not have discriminant of 16?

    a. $x^2 + 4x = 0$
    b. $x^2 + 2x - 3 = 0$
    c. $4x^2 + 8x + 3 = 0$
    d. $2x^2 - 2 = 0$
    e. $x^2 - 16 = 0$

26. If the lines formed by $6x - by = 18$ and $-2x + 5y = 7$ intersect at the point $(4, c)$, find bc.

    a. 3   b. -2   c. 12   d. 6   e. -30

27. What shape is formed by the equation $3y^2 + 3x + 1 = 4y - 4x^2 + 2$?

    a. circle   b. hyperbola   c. ellipse   d. parabola   e. No points satisfy the equation

28. Determine the distance between (4,2) and (-11,7).

    a. $3\sqrt{34}$ b. $\sqrt{74}$ c. $\sqrt{130}$ d. $5\sqrt{10}$ e. $2\sqrt{13}$

29. What is the real valued component of $(4 + i)^3$?

    a. 4   b. 16   c. 52   d. 60   e. 64

30. Which of the following statements are true for all real values of x?

    I. $\sqrt{|x|} > 0$ II. $\ln|x| > 0$ III. $\pi^x > 0$

    a. I, II, and III
    b. I and II only
    c. I and III only
    d. I only
    e. III only

31. Which of the following functions has a y-intercept of 2, a x-intercept of 2, and approaches -1 as x approaches infinity?
    a. $f(x) = 3^{-x} - 1$
    b. $g(x) = 3 \times 2^{-x} - 1$
    c. $h(x) = 3 \times \left(\sqrt{3}\right)^{-x} - 1$
    d. $j(x) = 3^x - 2x - 1$
    e. $k(x) = 3^{-2x} - 1$

32. What is the remainder when $4x^4 - 2x^3 - 3x - 5$ is divided by $x^2 + 2$?
    a. 81   b. $4x + 17$ c. $-x + 22$ d. $x + 11$ e. $-43x - 5$

33. A rectangular room has a length half the room's height, a width one foot less than the room's height, and a measure along the floor from one corner to the opposite corner (along the floor diagonal) one foot more than the room's height. Find the volume of the room. (all units are ft^3 for the answers below:
    a. 1,920   b. 2,040   c. 2,176   d. 4,800   f. 32,640

34. If the equation $y = x^2 + 6x - 7$ were rewritten in the form $y = a(x - h)^2 + k$ for appropriate values of a, h, and k, which of the following expressions is largest?
    a. ahk b. k^{h^a} c. $a + h + k$ d. $a - h - k$ e. $\dfrac{\frac{k}{h}}{a}$

35. A line passes through the points (2,3) and (8,8). If the line also passes through $(a, 5.5)$, find the value of a.
    a. $5.\overline{3}$ b. 5   c. $4.\overline{6}$ d. 4.5   e. $2\sqrt{5}$

36. A jar contains 10 marbles, 6 red and 4 black. A second jar contains 5 red marbles and an unknown number of black marbles. If a single marble is drawn from each jar, the probability that both are red is ¼ . How many black marbles are in the second jar?
    a. 7   b. 4   c. 5   d. 10   e. 2