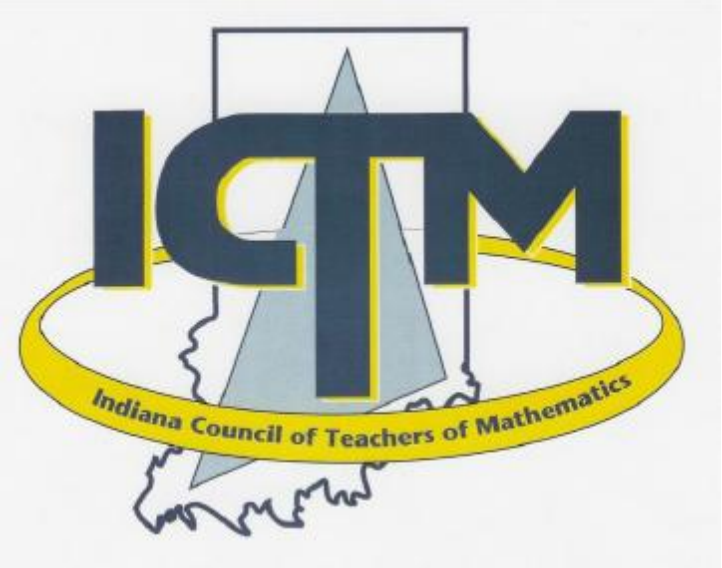


Comprehensive 2018

Sponsored by the Indiana Council of Teachers of Mathematics

Indiana State Mathematics Contest

This test was prepared by faculty at **Ball State University**

ICTM Website

http://www.indianamath.org/

Do not open this test booklet until you have been advised by the test proctor.

Next year's math contest date:

1. The function f has domain $5 \le x \le 7$ and range $-2 \le f(x) \le 4$.
   What are the domain and range, respectively, of the function $g(x) = 4 - 3f(2x+1)$?

   (A) $D: 2 \le x \le 3$; $R: 0 \le g(x) \le 2$
   (B) $D: 8 \le x \le 12$; $R: 0 \le g(x) \le 2$
   (C) $D: 2 \le x \le 3$; $R: -8 \le g(x) \le 10$
   (D) $D: 8 \le x \le 12$; $R: -8 \le g(x) \le 10$
   (E) None of the above

2. Last week, Dr. Doggie was murdered at his home in Muncie, Indiana. The police were able to place the time of his death between 11:10pm and 11:30pm. They arrested 4 suspects: Miss Kittie, Professor Python, Captain Kangaroo, and Timid Turtle. The 4 suspects made the following statements to the police:

   Ms. Kittie: I did not do it. Timid Turtle did it. Dr. Doggie was blackmailing Timid Turtle. Professor Python and I were watching television together from 10:10pm until 12:30am.

   Prof. Python: I'm innocent. Miss Kittie and I were watching television at the time of the murder. Timid Turtle was being blackmailed by Dr. Doggie. I saw Timid Turtle speaking to Dr. Doggie at 9:30pm on the night of the murder.

   Capt. Kangaroo: I'm innocent. Timid Turtle was being blackmailed by Dr. Doggie. Miss Kittie murdered Dr. Doggie. I saw Timid Turtle leave the house at 10:00pm.

   Timid Turtle: I did not kill Dr. Doggie. I was not being blackmailed by Dr. Doggie. I was in Chicago during the entire night of the murder. Professor Python murdered Dr. Doggie.

   If each of the 4 suspects made exactly 2 true statements and told exactly 2 lies, then who is the murderer?

   (A) Miss Kittie
   (B) Professor Python
   (C) Captain Kangaroo
   (D) Timid Turtle
   (E) Dr. Doggie

3. The owner of a pet shop bought a certain number of hamsters and half that many pairs of parakeets. He paid \$2 for each hamster and \$1 for each parakeet. For each pet he charged a price that was 10 percent more than what he paid for it.

   After all but 7 of the pets had been sold, the owner found that he had received an amount of money exactly equal to what he had originally paid for all of them. His potential profit, therefore, was represented by the combined value of the 7 remaining animals. What was this value?

   (A) \$11.00 (B) \$12.10 (C) \$13.20 (D) \$14.30 (E) None of the above

4. An engineer, noted for her ability to visualize 3-dimensional structure, was having coffee and doughnuts. Before she dropped a sugar cube into her cup, she placed the cube on the table and thought:

   > If I slice a horizontal plane vertically through the center of a cube, the cross section will be a square.
   > If I slice it through the center and 4 corners of the cube, the cross section will be a rectangle.
   > Now suppose I slice it this way with the plane… then the cross section is a regular hexagon.

   If the length of each side of the cube is half an inch, then what is the area of this regular hexagon?

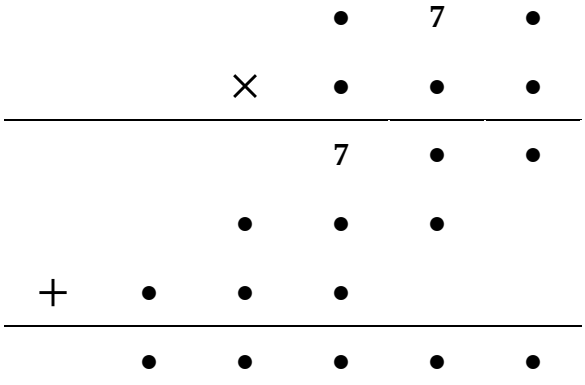
   (A) $\frac{2\sqrt{2}}{32}$ (B) $\frac{3\sqrt{3}}{16}$ (C) $\frac{\sqrt{2}}{4}$ (D) $\frac{3\sqrt{3}}{4}$ (E) None of the above

5. Bill and Ted flip one fair coin every day to see who will pay for lunch. One day Bill says: "Since I have won the last 3 coin flips and you have paid for the last 3 lunches, I will give you a break today. You can flip 2 fair coins and I will flip 1. If you have more heads than I have, then you win and I will pay for lunch. Otherwise, I win and you will pay for lunch again."

   When only 1 fair coin was used, Ted's probability of winning was $1/2$.
   Under the new arrangement, what is the probability that Ted will win and not have to pay for lunch again?

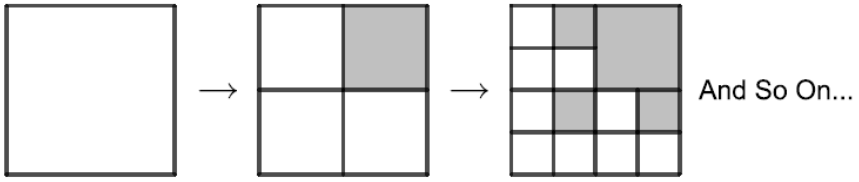
   (A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$ (E) None of the above

6. Each dot ● in this multiplication-by-hand problem represents a decimal digit from 0 to 9, inclusive. Each digit from 0 to 9 appears exactly 2 times each. In order to provide you with some good luck, the two 7s are already given to you. What is the 5-digit decimal number represented by the bottom 5 dots? In other words, what is the answer to this multiplication-by-hand problem?



   (A) 19233 (B) 35665 (C) 40096 (D) 55028 (E) None of the above

7. Begin with a square with side length of 1. Divide this square into 4 equal smaller squares and shade the upper-right of these squares. Then do the same with each of the 3 unshaded squares. Continue in this manner forever. What is the limit of the total shaded area of the original square?



(A) $\frac{2}{3}$ (B) $\frac{3}{4}$ (C) $\frac{4}{5}$ (D) $\frac{5}{6}$ (E) None of the above

8. The parabola that passes through the points $(-1,\ 247)$, $(1,\ 151)$, and $(2,\ 112)$ attains its minimum value when x equals:

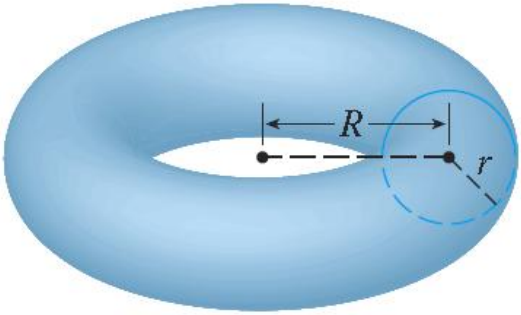
(A) 7 (B) 8 (C) 9 (D) 10 (E) None of the above

9. On December 26, 2017, Jonathan Pace, using open-source software known as GIMPS, discovered the 50th known Mersenne Prime. This number, which can be written as $2^{77,232,917}-1$, is now the largest prime number known to humans. It is 910,807 digits longer than the previous record prime number!

   Given that the number $2^{77,232,917}-1$ has the same number of digits as the number $2^{77,232,917}$, exactly how many digits does the largest prime number known to humans have?

(A) 23,249,425 (B) 53,533,779 (C) 177,835,364 (D) 256,562,197 (E) None of the above

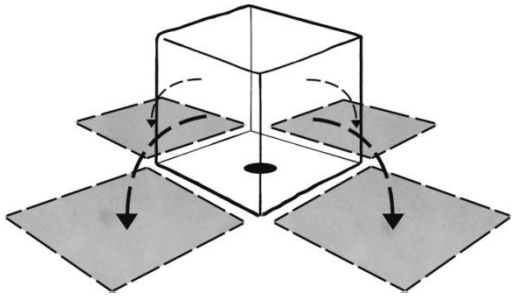
10. Begin with a circle of radius r centered at the point $(R, 0)$ on the x-axis (with $R > r$). Rotate this circle around the y-axis. This mathematical object is called a Torus and is shaped like an inner tube or a doughnut. Find the Surface Area of a Torus with $R = 5$ and $r = 2$.



(A) 29π (B) 40π (C) $29\pi^2$ (D) $40\pi^2$ (E) None of the above

11. The following question is from an exam used to help select who has what it takes to become an astronaut:

Imagine that you are facing a cube with a dot on the bottom face. The cube can roll to the left, right, towards you, or away from you. Now in your mind roll the cube: towards you, left, left, towards you, right, away from you, right. Where is the dot?



(A) On The Face Towards You
(B) On The Left Face
(C) On The Right Face
(D) On The Face Away From You
(E) None of the above

12. Consider the infinite triangle below:

1

2 3

4 5 6

7 8 9 10

11 12 13 14 15

16 17 18 19 20 21

⋮ ⋮ ⋮ ⋮ ⋮

And So On…

For each number in this infinite triangle let R be its row and P be its position in that row. For example, the number 14 in the infinite triangle above has (R, P) = (5, 4) and the number 17 has (R, P) = (6, 2).

What is the value of R + P for the number 12345678910?

(A) 157,135 (B) 210,500 (C) 258,105 (D) 308,420 (E) None of the above

13. The smallest 5-digit integer is 10,000. The largest 5-digit integer is 99,999.
What is the sum of all of the even 5-digit integers?

(A) 2,474,955,000
(B) 2,475,000,000
(C) 2,949,955,000
(D) 2,999,910,000
(E) None of the above

14. If $2\ln(x-2y)=\ln(x)+\ln(y)$, then what is the value of $\frac{x}{y}$?

(A) 1 only (B) 4 only (C) 6 only (D) 8 only (E) None of the above

15. An urn contains marbles of 4 solid colors: Red, White, Blue, and Green. When 4 marbles are randomly selected without replacement, each of the following events are equally likely:
(a) selecting 4 Red marbles
(b) selecting 1 White and 3 Red marbles
(c) selecting 1 White, 1 Blue, and 2 Red marbles
(d) selecting 1 Red, 1 White, 1 Blue, and 1 Green marble
What is the smallest number of marbles that the urn must contain to satisfy all of these conditions?

(A) 19 (B) 21 (C) 46 (D) 69 (E) None of the above

16. Let $\mathrm{A}=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\vec{x}=\begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $\vec{y}=\begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\vec{z}=\begin{bmatrix} 4 \\ 2 \end{bmatrix}$.
If $\mathrm{A}\vec{x}=\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\mathrm{A}\vec{y}=\begin{bmatrix} 8 \\ -3 \end{bmatrix}$, and $\mathrm{A}^{-1}\vec{z}=\begin{bmatrix} \alpha \\ \beta \end{bmatrix}$, then what is the value of $2\alpha+\beta$?

(A) 1 (B) 5 (C) 6 (D) 8 (E) None of the above

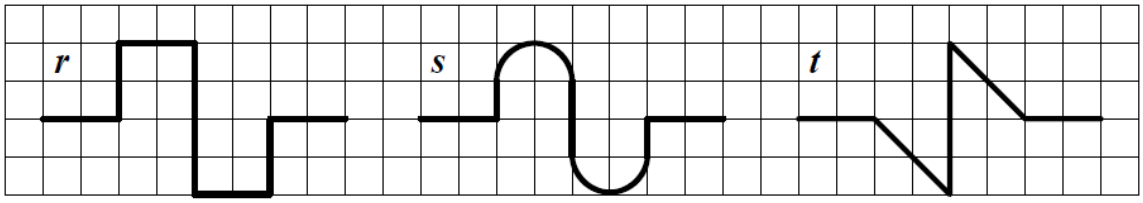
17. If $i^2=-1$, then what is the value of $\sqrt[3]{2+11i}+\sqrt[3]{2-11i}$?

(A) –4 (B) 0 (C) 4 (D) 11 (E) None of the above

18. The region defined by the inequality $|3x-18|+|2y+7|\le 3$ is a quadrilateral in the xy-plane.
What is the sum of all of the x-coordinates and y-coordinates of the 4 vertices of this quadrilateral?

(A) –2 (B) 10 (C) 14 (D) 20 (E) None of the above

19. Let *r*, *s*, and *t* denote the lengths each of the 3 lines in the following picture. Which of the inequalities is correct?



(A) $r<s<t$ (B) $t<r<s$ (C) $s<r<t$ (D) $s<t<r$ (E) None of the above

20. In the grid below each row, column, and 3×3 box must contain exactly one of each digit from 1 to 9. What is the value of the digit X?

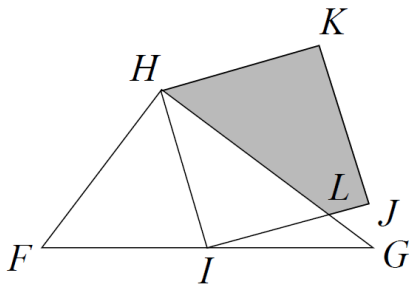
	1			8			4	
5					6			1
			9		7			
	6	4				7		
9					X			6
		7				8	5	
			8		9			
6			4					2
	3			7			6	

(A) 1 (B) 2 (C) 4 (D) 8 (E) None of the above

21. The 6-sided die below has the property that each pair of numbers on opposite faces has the same sum. The numbers on the hidden faces are all prime numbers. Which number is opposite to the 14 shown?

(A) 11 (B) 13 (C) 17 (D) 19 (E) None of the above

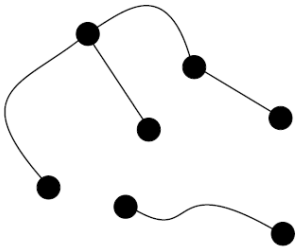
22. Triangle FHG has $FH = 6$, $GH = 8$, and $FG = 10$. The point I is the midpoint of FG, and $HIJK$ is a square. HG and IJ intersect at point L. What is the shaded area of the quadrilateral $HLJK$?



(A) $\frac{124}{8}$ (B) $\frac{125}{8}$ (C) $\frac{126}{8}$ (D) $\frac{127}{8}$ (E) None of the above

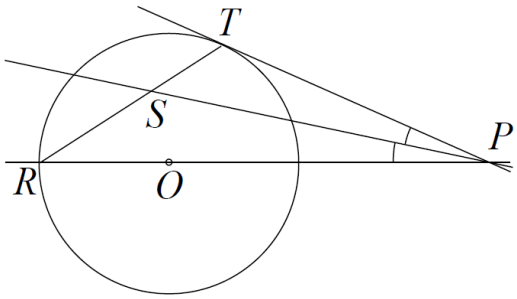
23. The graph below shows 7 vertices (the dots) and 5 edges (the lines) connecting them. An edge here is defined to be a line that connects 2 vertices together. In other words, an edge cannot loop back and connect to the same vertex. Remember that edges are allowed to cross each other. Also remember that the crossing of 2 edges does not create a new vertex. There will always be exactly 7 vertices in this problem, regardless of the number of edge crossings.

What is the least number of edges that could be added to the graph, in addition to the 5 already present, so that each of the 7 vertices has the same number of edges?



(A) 5 (B) 7 (C) 9 (D) 11 (E) None of the above

24. PT is tangent to the circle with center O and PS is the angle bisector of angle RPT. What is the measure of angle TSP?



(A) 42°
(B) 43°
(C) the angle depends on the position of point P
(D) 44°
(E) None of the above