

**Indiana Council of Teachers of Mathematics**  
**State Mathematics Competition**  
**Pre-Algebra 2023**

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### Answers

1. Correct answer: b

Solution: If the original rectangle had area  $A = \ell w$  then by increasing the length by 10% and the width by 20% we have  $A_{\text{new}} = (1.1 \cdot \ell) \cdot (1.2 \cdot w) = 1.32 \cdot \ell w$  which is 32% larger than the original.

2. Correct answer: b

Solution: The largest four numbers are: 872, 827, 782, 728, hence  $872 - 728 = 144$ .

3. Correct answer: c

Solution: If Jane is  $x$  years old, the other siblings are  $x - 2$  and  $x + 3$  years old. Sum of their ages is  $3x + 1 = 25$ , i.e.  $x = 8$ .

4. Correct answer: d

Solution:  $\frac{3}{0.07} - \frac{2}{0.05} + \frac{1}{0.35} = 3/(7/100) - 2/(5/100) + 1/(35/100) = \frac{300}{7} - \frac{200}{5} + \frac{100}{35} = \frac{1500 - 1400 + 100}{35} = \frac{200}{35} = \frac{40}{7}$  which is a reduced fraction as 40 is not divisible by 7. Hence, for  $nx$  to be an integer  $n$  must be divisible by 7, and 7 is the only such number among the given ones.

5. Correct answer: d

Solution: Each octagon has 8 sides, thus 24 octagons have  $24 \cdot 8 = 192$  sides. With 192 sides, we can make  $192/3 = 64$  triangles since a triangle has 3 sides.

6. Correct answer: b

Solution:  $\pi$  is approximately 3.14, and the circular arc that Claire walks is thus  $\frac{3}{4} \cdot 2 \cdot 3.14 = 4.71$  miles long. Since the distance from  $A$  to  $B$  and  $B$  to  $C$  is 1 mile each, the total length of the walk is 6.71 miles.

7. Correct answer: d

Solution: Sam takes 15 minutes to go from his house to the store and back (excluding the 10 minutes spent in shopping). If  $d$  miles is the distance between his house and the store, time taken to go to the store is  $d/4$  hr and time taken to go back is  $d/2$  hr. Therefore,  $d/4 + d/2 = 15/60$ . Solving this gives  $d = 1/3$  miles.

8. Correct answer: a

Solution:  $(1 + \frac{1}{3})(1 + \frac{1}{5})(1 + \frac{1}{7})(1 + \frac{1}{9}) = \frac{4}{3} \cdot \frac{6}{5} \cdot \frac{8}{7} \cdot \frac{10}{9} = \frac{4 \cdot 2 \cdot 8 \cdot 2}{7 \cdot 9} = \frac{2 \cdot 8^2}{7 \cdot 9}$ . Hence for  $\frac{2 \cdot 8^2}{7 \cdot 9} \cdot x^2 = \frac{2}{7}$  we must have  $x^2 = \frac{9}{8^2}$  and thus  $x = \frac{3}{8}$ .

9. Correct answer: c

Solution: Given five ingredients, call them here  $a, b, c, d, e$ , there are 10 ways to choose three of them, namely:  $\{a, b, c\}, \{a, b, d\}, \{a, b, e\}, \{a, c, d\}, \{a, c, e\}, \{a, d, e\}, \{b, c, d\}, \{b, c, e\}, \{b, d, e\}, \{c, d, e\}$ .

10. Correct answer: b

Solution:  $\sqrt{z} = \sqrt{10} < 4$ ,  $\frac{z^2}{1+z} = \frac{100}{11} > 9$ ,  $\frac{1+z}{z} = \frac{11}{10} = 1.1$ ,  $\frac{20}{z+2} = \frac{20}{12} < 2$ ,  $\frac{1}{z} = \frac{1}{10} = 0.1$ .

11. Correct answer: e

Solution:  $(1.1)^4 = 1.4641$ , therefore they pay 46.41% extra.

12. Correct answer: d

Solution: If the number 1 is the leading digit, when the other three can only be 122, 212, and 221. Similarly, if 2 is the leading digit, then the other three can only be 112, 121, and 211. Hence there are six such numbers.

13. Correct answer: c

Solution: Since the area of the cross-section is  $3\pi$ , we know that the radius is  $\sqrt{3}$ . Plugging that into the volume formula, we can arrive at the volume of  $4 \cdot \pi \cdot \frac{1}{3}\sqrt{3}^3 = 4 \cdot \pi \cdot \sqrt{3} = 21.77 \text{ in}^3$ .

14. Correct answer: a

Solution: Integers divisible by 7: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98.

Note that 21, 42, 63, 84 are divisible by 3.

This leaves us with 7, 14, 28, 35, 49, 56, 70, 77, 91, 98.

15. Correct answer: d

Solution: Sum of the first five scores is  $95 \cdot 5 = 475$ . We want the average of the six scores to be 90, i.e. the sum of the six scores should be  $90 \cdot 6 = 540$ . Score needed on the sixth test is thus  $540 - 475 = 65$ .

16. Correct answer: a

Solution: In order for  $1A3 + 876$  to be a 4-digit number one must have  $A + 7 \geq 10$ , in which case  $1A3 + 876 = 10R9$ , where  $A + 7 = 10 + R$ . Hence  $R = B$ . We are looking for the largest  $A$  such that  $10B9 = 10R9$  is divisible by 3. We try  $A = 9$ , which gives  $R = 6$ , but the sum of the digits of  $10R9 = 1069$  is 16, which is not divisible by 3. Then we try  $A = 8$ , which gives  $R = 5$ , and the sum of the digits of  $10R9 = 1059$  is 15, which is divisible by 3, and so is 1059. Hence  $A = 8$  is the sought-for number.

17. Correct answer: c

Solution: Using the midpoint formula we can see that  $\frac{7+x}{2} = 9$  and  $\frac{y+3}{2} = -2$ , which when solved give  $x = 11$  and  $y = -7$ . Then we have  $11 - (-7) = 18$ .

18. Correct answer: e

Solution: One solves for  $y$  and obtains  $y = \frac{2x-4}{14} = \frac{x-2}{7}$ . Hence  $x - 2$  must be divisible by 7.

19. Correct answer: d

Solution: Suppose the bag contains  $4x$ ,  $5x$  and  $10x$  coins of  $25\text{¢}$ ,  $10\text{¢}$ , and  $5\text{¢}$ , respectively. The value of the bag is  $4x \cdot (0.25) + 5x \cdot (0.10) + 10x \cdot (0.05) = x \cdot (1 + 0.5 + 0.5) = x \cdot 2$  dollars, which is, by assumption, 50. Therefore  $x = 25$ , and there are  $5x = 125$  coins of  $10\text{¢}$  in the bag.

20. Correct answer: a

Solution: The inequality  $x + y < x^2$  is equivalent to  $y < x^2 - x$ . If  $x \leq 1$ , then  $x^2 \leq x$ , hence  $x^2 - x \leq 0$ , which is not possible since  $y < x^2 - x$  and  $y > 0$ . None of the other inequalities must be true. (Since  $3 + 4 < 3^2$  the inequality  $y < 1$  (b) must not be true, the inequality  $x < 1$  (c) must not be true, the inequality  $y < x$  (d) must also not be true. And because  $2 + \frac{1}{2} < 2^2$ , the inequality  $1 < y$  must not be true.)

21. Correct answer: b

Solution: A linear pair forms a  $180^\circ$  angle. We know that one angle is  $38^\circ$  larger than the other. We can write an equation to solve:  $x + (x + 38) = 180$  and this gives us our smaller angle of  $71^\circ$ .

22. Correct answer: b

Solution: Notice the denominators are the sequence of squares. Meaning the sequence can be written,  $1, \frac{1}{2^2}, \frac{1}{3^2}, \frac{1}{4^2}, \frac{1}{5^2}, \frac{1}{6^2}, \frac{1}{7^2}, \frac{1}{8^2}, \frac{1}{9^2}, \frac{1}{10^2}$ . So the "... " represent the terms  $\frac{1}{5^2}, \frac{1}{6^2}, \frac{1}{7^2}, \frac{1}{8^2}$ .

23. Correct answer: c

Solution: Height of one book is  $8/48 = 1/6$  feet or 2 inches. So a 16 inches high stack has 8 books.

24. Correct answer: a

Solution: If Tom has saved  $\$x$ , then  $18/x = x/8$ , hence  $x^2 = 8 \cdot 18 = 144$ , and thus  $x = 12$ .

25. Correct answer: d

Solution: Using the multiplication principle we have  $3 \cdot 4 \cdot 2 \cdot 4 = 96$  options.

26. Correct answer: e

Solution: 105 has prime factorization  $3 \cdot 5 \cdot 7$ . So we have the set of divisors  $\{1, 3, 5, 7, 3 \cdot 5, 3 \cdot 7, 5 \cdot 7, 3 \cdot 5 \cdot 7, 105\} = \{1, 3, 5, 7, 15, 21, 35, 105\}$

27. Correct answer: b

Solution: With the same quantity of food, the farmer can feed 1 animal for  $20 \times 8 = 160$  days (less animals, more days). So he can feed 32 animals for  $\frac{20 \times 8}{32} = 5$  days.

28. Correct answer: c

Solution:  $AB$  is at least 10. And  $AB$  cannot be 90 or greater, because then  $CD = 99 - AB$  would be at most 9. Hence  $AB$  can take all values between 10 and 89, which gives  $89 - 10 + 1 = 80$  possible numbers for  $AB$ . Note that  $CD$  is automatically determined by  $AB$ , and  $CD \neq AB$ , because 99 is odd.

29. Correct answer: a

Solution: If 7 apples are green and there are 15 total apples, the probability of randomly choosing a green apple is 7 out of 15, hence  $7/15$ .

30. Correct answer: d

Solution: We have  $y = x + 1$  and  $z = x + 2$ , hence  $x + y + z = 3x + 3 = 3(x + 1)$ . Of the given numbers only 102 is divisible by 3,  $102 = 3 \cdot 34$ , and we have  $x = 33$  in this case.

31. Correct answer: c

Solution: Let  $x$  be the smallest integer and  $x + 6$  be the largest integer in the set. Since 6 is the only mode, it appears at least twice. Let the remaining integer be  $y$  (i.e., different from  $x$ ,  $x + 6$  and 6. Since 6 is the median, the only choices for the increasing order of the integers in the set is  $x, y, 6, 6, x + 6$  or  $x, 6, 6, y, x + 6$ . As 6 is the mean, the sum of the five integers is  $6 \times 5 = 30$ , i.e.  $x + 6 + 6 + y + x + 6 = 30$  or  $2x + y = 12$ . Being the smallest integer, the only possibilities for  $x$  are  $x = 1, 2, 3, 4, 5, 6$ . If  $x = 6$ , then  $y = 0$ , which is not possible as the integers are positive. If  $x = 5$ , then  $y = 2$ , which is not possible as  $x$  is the smallest integer. If  $x = 4$ , then  $y = 4$ , which is not possible as 6 is the only mode. If  $x = 2$ , then  $y = 8$  and the largest term is also  $x + 6 = 8$ , which is not possible as 6 is the only mode. Finally, if  $x = 1$ , then  $y = 10$ , but the largest term is  $x + 6 = 7$ , a contradiction. Therefore,  $x = 3$  is the only possible choice. This makes the smallest term 3 and the largest term 9, so that their sum is 12.

32. Correct answer: e

Solution: There are 50 even integers between 2 and 100:  $2 \cdot 1, 2 \cdot 2, 2 \cdot 3, \dots, 2 \cdot 50 = 100$ . If we subtract of each of these 1, we obtain the odd integers between 1 and 99:  $2 \cdot 1 - 1 = 1, 2 \cdot 2 - 1 = 3, 2 \cdot 3 - 1 = 5, \dots, 2 \cdot 50 - 1 = 99$ . Therefore, the sum of the even integers between 2 and 100 (including 2 and 100) exceeds the sum of the odd integers between 1 and 99 (including 1 and 99) by 50.

33. Correct answer: d

Solution: We have  $A = \frac{1}{2}bh = 6$ . By Pythagoras' theorem the hypotenuse is  $\sqrt{3^2 + 4^2} = \sqrt{25} = 5$ . This gives  $P = 12$  and hence  $2A + P = 24$ .

34. Correct answer: b

Solution:  $\frac{2}{a} + \frac{3}{b} = \frac{1}{c}$ . Multiply the equation by  $a \cdot b$  gives  $2b + 3a = \frac{a \cdot b}{c}$ . Substitute using  $ab = 384$  and  $c = 4$  gives  $2b + 3a = \frac{384}{4} = 96$ .

35. Correct answer: e

Solution: The effective cost at supermarket  $X$  amounts to  $0.90 \times 1.04 = 0.936$ , that means savings of 6.4%. The effective cost at supermarket  $Y$  amount to  $0.80 \times 1.17 = 0.936$ , that means savings of 6.4%. Hence there are no extra savings when shopping at supermarket  $Y$ .

36. Correct answer: a

Solution:  $y = \sqrt{4^2 - 6} = \sqrt{10}$  and thus  $x = \frac{21}{\sqrt{10}}$ . This gives  $x + y = \frac{21}{\sqrt{10}} + \sqrt{10} = 9.80$ .

37. Correct answer: c

Solution: Letting the radius of the small circle be  $r$ , the radius of the larger circle will be  $2r$ . This makes the area of the smaller circle  $\pi r^2$  and the area of the larger circle  $4\pi r^2$ .

38. Correct answer: b

Solution: Since a week has seven days, the eighth day, the 15th day, and so on, will be a Monday if the first day that we count is a Monday. We have  $100 = 98 + 2 = 14 \cdot 7 + 2$ . Hence the 99th day is a Monday too, and the 100th day is a Tuesday.

39. Correct answer: e

Solution: Of the animals had as pets, only dogs and cats have fur. Therefore, there are 38 out of 50 students that meet that criterion, and  $38/50 = 19/25$ .

40. Correct answer: e

Solution: Write  $a = 3a'$  and  $b = 3b'$  with coprime integers  $a'$  and  $b'$ . Then  $9a'b' = 36$ , hence  $a'b' = 4$ . If  $a' = 1$ , then  $b' = 4$  and  $a + b = 3(1 + 4) = 15$ . If  $a' = b' = 2$ , then  $a = b = 6$ , and the greatest common divisor would be 6.