

Indiana State Math Contest 2022 Geometry/Integrated II Exam

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Mark your calendar:

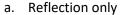
ICTM State Awards Ceremony 2022: Friday, June 3, 2022 ICTM State Math Contest 2023: Saturday, April 22, 2023

Do not open this test booklet until you have been advised to do so by the test proctor.

- For a certain platonic solid, there are x faces, y vertices, and z edges, where the minimum of these values is 6, and x + y + z = 26. What solid(s) could it be?

 a. A cube or an octahedron
 b. A cube or a dodecahedron
 c. A cube or an icosahedron
 d. Only a cube
 e. None of the above
- 2. Let *P* be an n-sided polygon with a perimeter of 24 cm. To the nearest tenth of a square centimeter, what is the maximum area *P* can enclose?
 - a. 45.8
 - b. 452.4
 - c. 183.3
 - d. 36.0
 - e. None of the above
- 3. Let T be a right triangle with hypotenuse length l and a non-right angle of θ . If $\sin \theta = \frac{7}{25}$ find the perimeter of
 - a. 2.24*l*
 - b. 56*l*
 - c. 2.5*l*
 - d. 2.82*l*
 - e. None of the above
- 4. On a sunny day, if the 4 foot 6-inch tall Lord Farquaad casts a 10 foot 6-inch shadow, then Shrek standing in the same spot would cast a 16 foot 11-inch shadow. How tall is Shrek?
 - a. 7 feet 2 inches
 - b. 7 feet 3 inches
 - c. 7 feet 4 inches
 - d. 7 feet 8 inches
 - e. None of the above
- 5. For what central angle is the measure of an arc equal to the radius of its circle? Round your answer to the nearest tenth of a degree.
 - a. 36.5°
 - b. 47.2°
 - c. 57.3°
 - d. 60.0°
 - e. None of the above

6. Which transformation of the plane would take the grey trapezoid to the black trapezoid in a single step?

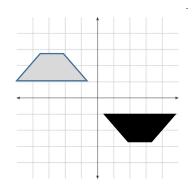


b. Translation only

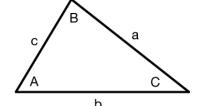
c. Rotation only

d. Either a Reflection OR Rotation

e. Either a Rotation OR Translation



7. The statement "Two triangles with a common angle measure, common side length adjacent to the angle, and common side length opposite the angle are congruent" is false. A counterexample would be an example where the given information could be used to create two different triangles. Of the options below, which could be used to create the counterexample?



a. $\angle A=105^{\circ}$, |a|=6 in., |b|=4 in.

b. $\angle A=90^{\circ}$, |a|=13 in., |b|=5 in.

c. $\angle A=30^{\circ}$, |a|=4 in., |b|=2 in.

d. $\angle A=45^{\circ}$, $|a|=\sqrt{2}$ in., |b|=1 in.

e. None of the above are counter examples

8. For a cube of volume 1, if the volume of the cube is doubled, by how many times does the surface area increase? Round your answer to the nearest tenth.

a. 7.6

b. 1.6

c. 9.5

d. 12.0

e. None of the above

9. According to South Carolina Law, "It is unlawful on an election day within two hundred feet of any entrance used by the voters to enter the polling place for a person to distribute any type of campaign literature or place any political posters." Further, legal precedent has established this to be "as the crow flies" from the entrance. If the entrance to a polling station is 100 feet north of the corner of Lye and Steel on the east side of Steel Street, then to the first legal foot, how far east down the north side of Lye Street must you go from that corner before you can place your first political poster?

a. 174 ft

b. 156 ft

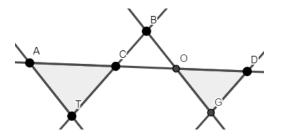
c. 142 ft

d. 100 ft

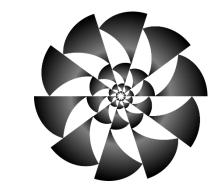
e. None of the above

10. If $\overline{AT}||\overline{BO}|$ and $\overline{BC}||\overline{DG}|$, then

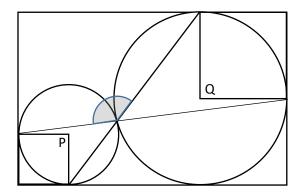
- a. Δ CAT is similar to Δ DOG
- b. Δ CAT is similar to Δ GOD
- c. \triangle ACT is similar to \triangle DOG
- d. $\triangle ACT$ is similar to $\triangle GOD$
- e. None of the above are correct



- 11. In the image to the right, the n-fold rotational symmetry could best be determined to be what value for n?
 - a. 20
 - b. 3
 - c. 10
 - d. 5
 - e. None of the above

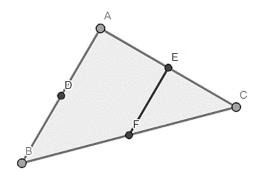


- 12. In the picture to the right, the circles centered at P and Q touch at exactly one point, and they touch the outer rectangle at exactly two and three points, respectively. Find the measure of the shaded angle.
 - a. 120°
 - b. 125°
 - c. 135°
 - d. 140°
 - e. None of the above



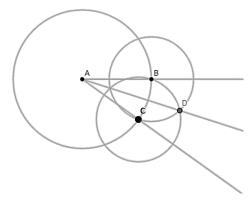
- 13. Consider the statement "If the four sides of a quadrilateral are congruent, then the quadrilateral is a rhombus." What is the contrapositive of this statement?
 - a. If the four sides of a quadrilateral are not congruent, then the quadrilateral is not a rhombus
 - b. If the four sides of a quadrilateral are congruent, then the quadrilateral is not a rhombus.
 - c. If the quadrilateral is a rhombus, then the four sides of the quadrilateral are congruent
 - d. If the quadrilateral is not a rhombus, then the four sides of the quadrilateral are not congruent.
 - e. None of the above
- 14. If two triangles in the same plane intersect each other, which of the following polygons could <u>not</u> be made by their intersection?
 - a. Triangle
 - b. Hexagon
 - c. Square
 - d. Octagon
 - e. All of the above could be made

- 15. If the interior angle measure of regular polygon P is multiplied by the exterior angle of P, the result is 3200. How many sides does P have?
 - a. 15
 - b. 18
 - c. 20
 - d. 160
 - e. None of the above
- 16. A Brinley Hardy lawn roller is a 270 pound cylinder that is 24 inches wide and 18 inches in diameter. If it is rolled for one complete rotation on flat ground, how much total area will it cover? Round your answer to the nearest ten square inches.
 - a. 680 in²
 - b. 2710 in²
 - c. 3050 in²
 - d. 6110 in²
 - e. None of the above
- 17. If you know that a certain four sided polygon has two congruent perpendicular diagonals, what is the most specific thing you could say about the shape?
 - a. It is a Quadrilateral
 - b. It is a Parallelogram
 - c. It is a Rectangle
 - d. It is a Rhombus
 - e. It is a Square
- 18. A vertical cross section of a three dimensional object is a triangle, while a horizontal cross section of an object is a rectangle. What is the shape?
 - a. A cone
 - b. A rectangular pyramid
 - c. A triangular prism
 - d. A hexagonal pyramid
 - e. More than one of these is possible
- 19. In the picture to the right, which of the following would not tell you that $\overline{AB}||\overline{EF}||$?
 - a. $\overline{AD} \cong \overline{EF} \cong \overline{BD}$
 - b. ∠BAE≅∠CEF
 - c. E is the midpoint of \overline{AC} and F is the midpoint of $\overline{\overline{BC}}$
 - d. $\triangle ABC$ is similar to $\triangle EFC$
 - e. All of these would tell you that $\overline{AB}||\overline{EF}|$

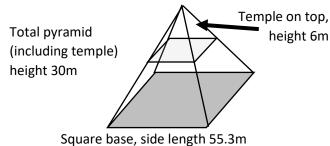


- 20. Find the perimeter of a triangle with vertices at (0,24), (-18,0) and (7,0).
 - a. 80
 - b. 75
 - c. 49
 - d. 66
 - e. None of the above

- 21. Consider the geometric construction to the right. Which of the following statements is **false**?
 - a. $2m(\angle DAB) = m(\angle CAB)$
 - b. $m(\angle ABC) = 2m(\angle BAD)$
 - c. $|\overline{AC}| = |\overline{AB}|$
 - d. ΔDBC is an isosceles triangle
 - e. None of the above



- 22. Little Nero's Pizza sells individual 8-inch pizzas, regular 12-inch pizzas, and family size 16-inch pizzas (all measurements are diameters). If every slice of pizza is cut to the same area, regardless of which pizza you take a slice from, and if the individual pizza is cut into 4 slices, how many slices should the regular pizza be cut into?
 - a. 6
 - b. 8
 - c. 9
 - d. 12
 - e. None of the above
- 23. Let Δ LFT be a right triangle, with the right angle at L. If you drop an altitude from L to the hypotenuse at point E, then the length of \overline{LE} is 4 units and the length of \overline{ET} is 8 units. What is the area of Δ LFT (in square units)?
 - a. 10
 - b. 32
 - c. 20
 - d. 16
 - e. None of the above
- 24. Let x, y and z be real numbers such that 9 > x > y > 1. Let T_x be a triangle with side lengths 4, 5, x, and T_y be a triangle with side lengths 4, 5, y. If the angle X is opposite the side of length x in T_x , and the angle Y is opposite the side of length y in T_y , which of the following statements must be true?
 - a. Y>X
 - b. The area of T_x must be greater than the area of T_y .
 - c. The area of T_y must be greater than the area of T_x .
 - d. $X+Y=180^{\circ}$
 - e. None of the above
- 25. The Mayan Pyramid of Kukulcan can be approximated by a right square pyramid with base sides of length 55.3m and a height of 30m. If the Temple only takes up the top 6m in height of the entire pyramid, what approximately is the volume of the temple?
 - a. 61 m³
 - b. 245 m³
 - c. 6116 m³
 - d. 170 m³
 - e. None of the above



- 26. Two lines are intersected by a transversal. Which of the following properties would ensure that the lines are parallel?
 - a. Alternate interior angles are congruent
 - b. Vertical angles are congruent
 - c. Corresponding angles are supplementary
 - d. Same-side interior angles are congruent
 - e. None of the above
- 27. In three dimensional space, what is the distance from the point (3, -4, 12) to the origin?
 - a. 11
 - b. $11\sqrt{3}$
 - c. $6\sqrt{5}$
 - d. 19
 - e. None of the above
- 28. If \overrightarrow{AB} is contained in plane \mathcal{P} , and \overrightarrow{AB} is perpendicular to plane \mathcal{R} , which of the following statements is true?
 - a. Plane \mathcal{P} is perpendicular to \overrightarrow{AB} .
 - b. Either point A or B must lie in plane \mathcal{R} .
 - c. Plane \mathcal{P} is parallel to plane \mathcal{R} .
 - d. Plane \mathcal{P} is perpendicular to plane \mathcal{R} .
 - e. None of the above
- 29. Find the center of a circle that passes through the points (2,1), (0,5), and (-1,2)
 - a. (-5,0)
 - b. (1.5,4.5)
 - c. (1,3)
 - d. $(0,3\frac{1}{3})$
 - e. None of the above
- 30. Consider the claim "The triangle with vertices at A(-2, -4), B(2, -1), and C(8, -9) is a right triangle." Is this claim correct, and if so, which angle is the right angle?
 - a. Yes, Vertex A
 - b. Yes, Vertex B
 - c. Yes, Vertex C
 - d. Cannot be determined
 - e. The triangle is not a right triangle
- 31. Looking to bring a new line of packing products to the market, the Premium Packing Corporation is interested in creating a new designer box for high end clients to purchase. To fit with customer needs, the boxes must have a volume of 1000 cubic centimeters, have at least one flat side to rest on when placed on the ground, and use the minimum amount of material in creating the package as possible. Given these design guidelines, which shape should they consider using?
 - a. Rectangular prism
 - b. Square pyramid
 - c. Cone
 - d. Cylinder
 - e. Sphere

- 32. Consider a parallelogram *ABCD*. Without being given any additional information, what triangle congruence property could you use to prove that $\triangle ABC \cong \triangle CDA$?
 - a. SSS
 - b. SAS
 - c. ASA
 - d. All three could be used
 - e. None of the above