Indiana State Math Contest
2022
Algebra I/Integrated I
Exam

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Mark your calendar:

ICTM State Awards Ceremony 2022: Friday, June 3, 2022

ICTM State Math Contest 2023: Saturday, April 22, 2023

Do not open this test booklet until you have been advised to do so by the test proctor.

1. Desi bought 3 pounds of apples at \$1.50/lb and 2½ pounds of oranges at \$1.80/lb. Her total was taxed at 8%. How much change did she receive from \$20?

A. \$9.72 B. \$9.36 C. \$11.00 D. \$9.00 E. \$10.28

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2. If the mean grade for Mr. Fibonacci's Geometry class of 30 students is 84 and the mean grade for Mr. Euclid's Geometry class of 25 students is 73, what is the mean of the grades for all 55 Geometry students?

A. 76.5 B. 77 C. 78.5 D. 79 E. 80

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3. During an appliance sale, a refrigerator priced at \$1880 was reduced to \$1700. What was the percent savings on the refrigerator, rounded to the nearest tenth of a percent?

A. 8.9% B. 9.0% C. 9.5% D. 9.6% E. 10.0%

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4. Find the absolute-value of the difference between the solutions for the absolute-value equation $|3x - 1| + 2 = 8$.

A. $\frac{1}{4}$ B. $\frac{7}{3}$ C. 5/3 D. 0 E. 4

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5. Let the rule for $a☺b$ be defined as $a☺b = \frac{a^2+b^2}{ab}$. Find $6☺2$.

A. 40 B. $\frac{10}{3}$ C. 5 D. $\frac{19}{6}$ E. 12

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6. Find the 71st term in the pattern 2, 6, 10, 14, …

A. 192 B. 196 C. 242 D. 282 E. 286

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7. Trent hits a golf ball from the second story of Top Golf. The height h of the ball in feet above the ground *t* seconds after it has been hit is given by $h(t) = -16t^2 + 50t + 25$. How long, in seconds, does it take for the ball to hit the ground? Round to the nearest hundredth of a second.

A. 3.67 seconds B. 1.85 seconds C. 3.56 seconds

D. 2.74 seconds E. 3.22 seconds

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8. Find the y-intercept of the line $\frac{3}{4}x - \frac{1}{2}y + 0.2 = \frac{5}{6} - 1.3x$

A. $(0, \frac{13}{15})$ B. $(0, -\frac{5}{3})$ C. $(0, -1.4)$ D. $(0, -\frac{19}{15})$ E. $(0, \frac{19}{30})$

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9. What is the *x*-intercept of the line containing (-4, 7) and (8, 11)?

A. (-25, 0) B. (9, 0) C. (25/3, 0) D. (-21, 0) E. (-23/9, 0)

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10. Find $2x + 5y$ if $\frac{7(x^3-2)}{2} + 3 = 220$ and $\frac{3}{2}(3y^5 + 2) - 1 = 146$.

A. 18 B. 6 C. 22 D. 28 E. 8

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11. Carson is making a cake for his younger brother. He has two choices of cake flavor (chocolate and vanilla), three colors of frosting (white, blue, and green), five colors of accent icing (red, yellow, orange, purple, and black). If he wants one cake flavor, one color of frosting, and two different colors of accent icing, how many possible cakes can be made? Suppose the accent combination of "red-yellow" is the same as the accent combination of "yellow-red."

A. 120 B. 60 C. 15 D. 30 E. 240

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12. Solve $\frac{1}{x^2} + \frac{1}{6x} = \frac{1}{6}$.

A. 1 and 6 B. -3 and 2 C. 2 and 3 D. -2 and 3 E. -6 and 1

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13. If $0 < j < k < 1$, which of the following expressions is the greatest?

A. jk B. $\frac{1}{j^2}$ C. $\frac{1}{k^2}$ D. j^2 E. k^2

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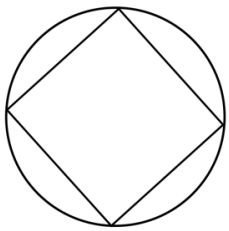
14. If the radius of the circle in the diagram below is 6 centimeters, find the perimeter of the inscribed square.



A. $6\sqrt{2}$ B. 24 C. $24\sqrt{2}$ D. 12 E. 72

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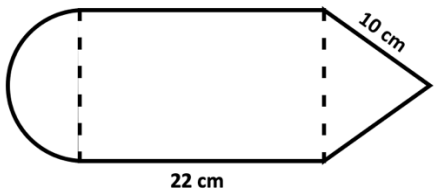
15. Find the area the composite shape comprised of an equilateral triangle with side length of 10 cm, a rectangle with width of 22 cm, and a semicircle. Round to the nearest tenth of a square centimeter.



A. 308.2 cm^2 B. 398.5 cm^2 C. 302.6 cm^2 D. 341.8 cm^2 E. 359.3 cm^2

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16. Find the solution set for the compound inequality $\frac{7}{6}x - \frac{3}{2} \geq \frac{5}{6}x$ and $\frac{1}{2}x + 0.8 < 2 + 0.3x$.

A. $-6 < x \leq \frac{9}{2}$ B. $\frac{4}{5} \leq x < 6$ C. $-6 < x \leq \frac{3}{4}$

D. $\frac{9}{2} \leq x < 6$ E. No solution

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17. Lori and Jonathan have both volunteered to make costumes for a middle school play. Lori has estimated that it would take her 20 hours to do the work alone. Jonathan has estimated that it will take him 30 hours working alone. To the nearest half hour, how long should it take them to make the costumes together?

A. 50 hours B. 25 hours C. 12 hours D. 11.5 hours E. 10 hours

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18. If a whole number *n* is picked at random from all whole numbers less than or equal to 9, what is the probability that $3n + 7 \leq 23$?

A. $\frac{3}{5}$ B. $\frac{5}{9}$ C. $\frac{1}{2}$ D. $\frac{6}{9}$ E. $\frac{4}{9}$

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19. Find the minimum *y*-value for $y = 2x^2 - 4x + 7$.

A. 7 B. -1 C. 1 D. -5 E. 5

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20. Simplify the expression: $\frac{(x^3y)(4x^{-1}y^2)}{12x^6y^0}$

A. $3x^4y^3$ B. $\frac{y^3}{3x^4}$ C. $\frac{x^4y^3}{3}$ D. $\frac{4y^2}{12x^4}$ E. $\frac{y^3}{48x^4}$

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21. A cylindrical paper towel roll is 11 inches long with a diameter of 2 inches. Shelby decides to use the cardboard for an art project and carefully cuts the roll, as shown by the dotted line below. When she flattens it, she has a rectangular piece of cardboard. Find the rectangle's area to the nearest square inch.

A. 71.7 in^2 B. 69.1 in^2 C. 66.5 in^2 D. 109.4 in^2 E. 138.2 in^2

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22. There are two solutions to the equation $6x^2 = 5x + 21$. Find the *mean* of the two solutions.

A. $-\frac{7}{12}$ B. $\frac{5}{12}$ C. $\frac{7}{3}$ D. $-\frac{3}{2}$ E. $-\frac{7}{4}$

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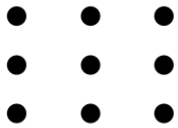
23. In the image below, how many triangles can be made by connecting any three dots to create the vertices of the triangle? (Note: The sides of the triangle might pass through other dots.)



A. 45 B. 36 C. 78 D. 84 E. 76

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24. Find the product of the two solutions of the equation $10x^2 - 11x + 3 = 0$.

A. $\frac{3}{10}$ B. $-\frac{3}{10}$ C. $\frac{3}{7}$ D. $-\frac{3}{7}$ E. $\frac{11}{20}$

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25. Solve the equation $\frac{3y^2-1}{2y^2+5} = x$ for y.

A. $y = \pm\frac{5x+1}{3-2x}$ B. $y = \pm\sqrt{\frac{5x+2}{3-x}}$ C. $y = \left(\frac{5x+1}{2x-3}\right)^2$

D. $y = \pm\sqrt{\frac{3x-1}{2x+5}}$ E. $y = \pm\sqrt{\frac{5x+1}{3-2x}}$

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26. A racecar driver practices for the Indianapolis 500 and completes 25 laps on the track. Based on her practice, she predicts she can travel at an average of 180 miles per hour for all 200 laps (including time for pit stops). If the race track is 2.5 miles around, how long would it take for her to finish a perfect race with no accidents, rounded to the nearest minute?

A. 2 hours and 7 minutes B. 2 hours and 38 minutes C. 2 hours and 47 minutes

D. 3 hours and 10 minutes E. 3 hours and 14 minutes

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27. Find the equation of a line that has the same value for its slope and *y*-intercept and passes through the point $(-7\ ,\ \frac{1}{3})$.

A. $y = -\frac{1}{6}x - \frac{1}{6}$ B. $y = 2x + 2$ C. $y = -\frac{1}{18}x - \frac{1}{18}$

D. $y = -2x - 2$ E. $y = -18x - 18$

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28. Suppose you have the fraction $\frac{a}{b}$. When 2 is added to the numerator, the resulting fraction is 1. When 3 is added to the denominator, the resulting fraction is $\frac{1}{2}$. What is the value of $b^2 - a^2$?

A. 5 B. 12 C. 24 D. 25 E. 28

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29. Hudson wants to order pizzas for his mathematics club. He sees a deal for four medium pizzas (14" diameter) for a total of $29.99 or three large pizzas (16" diameter) for $29.99. Which deal gives the mathematics club more pizza and by how many square inches? Round to the nearest tenth.

A. Four medium pizzas; 12.6 in^2 B. Four medium pizzas; 25.1 in^2

C. Three large pizzas; 12.6 in^2 D. Three large pizzas; 25.1 in^2

E. They have the same area

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30. The area of a square is 100 centimeters. A rectangle has the same perimeter as the square. The length of the rectangle is three times its width. What is sum of the lengths of the diagonals for the square and the rectangle?

A. $15\sqrt{2}$ B. $10\sqrt{2} + 5\sqrt{10}$ C. $20\sqrt{2}$ D. $25\sqrt{20}$ E. $10\sqrt{2} + 5\sqrt{6}$

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31. If *m* and *n* are positive integers and $2^m = 16^n$, what is the value of $\frac{m}{n}$?

A. 1/4 B. 4 C. 8 D. 1/8 E. cannot be determined

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32. Emma drives an average of 60 miles per hour for the 50 miles from Indianapolis to Bloomington. On her return trip, she gets stuck in construction and travels at an average rate of 40 miles per hour. What is her average speed for the full 100 miles?

A. 48.5 mi/hr B. 45 mi/hr C. 55 mi/hr D. 50 mi/hr E. 48 mi/hr

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33. Sharon uses 2.5 *ounces* of chocolate chips in each loaf of her chocolate chip banana bread. If she wants to make 12 loaves of bread for her friends, how many *pounds* of chocolate chips should she purchase?

A. 2 pounds B. 1.5 pounds C. 24 pounds D. 1.875 pounds E. 30 pounds

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34. Find the value of *m* that gives an infinite number of solutions to the system of equations

$$\begin{cases} y = \frac{2}{5}x - 2 \\ 20 + my = 4x \end{cases}$$

A. 10 B. -10 C. 5 D. -5 E. 8

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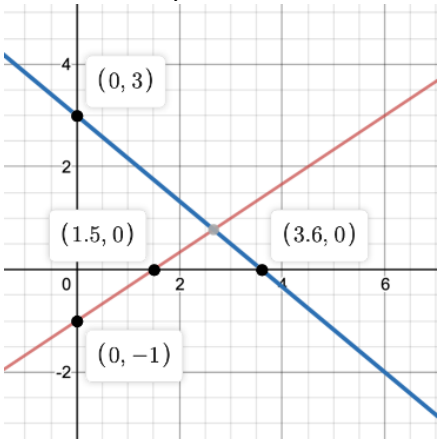
35. Line *m* has intercepts of (1.5, 0) and (0, -1) and line *n* has intercepts of (0,3) and (3.6, 0), as shown in the picture below. Find the *y*-value of the point of intersection.



A. $\frac{5}{6}$ B. $\frac{7}{9}$ C. $\frac{7}{8}$ D. $\frac{8}{9}$ E. $\frac{9}{10}$

36. Find the negative solution of $2\sqrt{b^2 + 6b - 3} + 2 = 10$

A. $-3 + 2\sqrt{7}$ B. $-3 + 4\sqrt{7}$ C. $-3 - 4\sqrt{7}$

D. $-3 - \sqrt{112}$ E. $-3 - 2\sqrt{7}$

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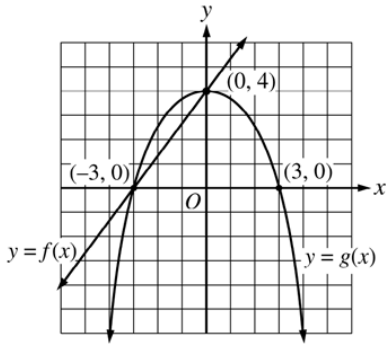
37. $f(x)$ and $g(x)$ are graphed in the image below and intersect at the points (-3, 0) and (0, 4). If $f(x)$ has the equation $f(x) = a(x + 3)(x - 3)$ and $g(x)$ has the equation $g(x) = mx + 4$, what is the sum of $a + m$?



A. $\frac{4}{3}$ B. $-\frac{4}{9}$ C. -3 D. 4 E. $\frac{8}{9}$

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38. The mean, median, and mode are equal in the set of numbers $\{20, M, 22, 8, M, 6, 9\}$. When 22 is removed from the set of numbers, what is the new mean?

A. $\frac{69}{7}$ B. $\frac{23}{2}$ C. 13 D. 13.5 E. $\frac{91}{6}$

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39. Ryan has an empty swimming pool that is 12 feet long and 16 feet wide. He wants to fill the pool to a depth of 4 feet. Instead of paying a pool company to fill his pool, he uses his garden hose, which pumps out water at 8 gallons per minute. There are 231 cubic inches in every gallon. How many hours will it take Ryan to fill his pool? Round to the nearest tenth of an hour.

A. 9.0 hours B. 10.2 hours C. 11.0 hours D. 11.8 hours E. 12.0 hours

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40. Assume all denominators are non-zero. Solve the equation: $\frac{2x-7}{x+3} = \frac{3x-2}{2x+8}$

A. x = -5 and x = 10 B. x = 5 and x = -10 C. x = -5 and x = 7

D. x = 10 and x = -7 E. x = -5 and x = -7