



Indiana State Math Contest

2022

Algebra II/Integrated III

Exam

This test was prepared by faculty of
University of Southern Indiana

Mark your calendar:

ICTM State Awards Ceremony: Friday, June 3, 2022

ICTM State Math Contest 2023: Saturday, April 22, 2023

Do not open this test booklet until you have
been advised to do so by the test proctor.

1. A rectangular garden has an area of 85 square meters and perimeter of 37 meters. What is the difference between the garden's length and width?

- a. $\frac{1}{2}$ m b. $1\frac{1}{2}$ m c. $3\frac{1}{2}$ m d. $6\frac{1}{2}$ m e. None of these
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2. On the first test in a class of 25 **students**, the male **students** averaged 70. When the female students' grades were added in, the class average rose to 75. If the females as a group averaged a whole number on the test, how many males are in the class?

- a. 5 b. 10 c. 15 d. 20 e. It cannot be determined
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3. Solve the inequality for x : $\frac{2x-3}{x+2} \geq 3x - 14$

- a. $(-\infty, -2) \cup [-\frac{5}{3}, 5]$
b. $(-\infty, -\frac{5}{3}] \cup (-\infty, 5]$
c. $(-\infty, 8] \cup (-\infty, 11]$
d. $[-\frac{5}{3}, 5]$
e. None of these
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4. Simplify $(2 - 3i)^4$

- a. $8 - 12i$ b. -65 c. $-119 + 120i$ d. $16 - 81i$ e. None of these
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5. A local store sells two kinds of candy bars. The price of the expensive kind is \$0.45 more than the price of the cheap kind. If the store sold 13 candy bars for a total of \$13.05, then how many expensive candy bars did the store sell?

- a. 1 b. 3 c. 5 d. 7 e. 9
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6. You select 3 marbles without replacement from a bag containing 3 red marbles, 2 blue marbles, and 1 white marble. What is the probability that your selection contains **exactly** two colors?

- a. $\frac{2}{3}$ b. $\frac{13}{20}$ c. $\frac{1}{2}$ d. $\frac{1}{3}$ e. None of these
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7. Exactly how many of the following statements are true?

- $a^x - a^y = a^{x-y}$
- $a^x \cdot a^y = a^{x \cdot y}$
- $\log_a x - \log_a y = \log_a(x - y)$
- $\log_a x \cdot \log_a y = \log_a(x \cdot y)$

- a. 0 b. 1 c. 2 d. 3 e. 4
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8. A manufacturer builds a machine that folds 500 envelopes in 8 minutes. They want to build a second machine so that, when working together, the two machines will fold 500 envelopes in 3 minutes. How fast will the second machine have to fold 500 envelopes?

- a. Just under 5 minutes
- b. Exactly 5 minutes
- c. Just over 5 minutes
- d. Exactly $5\frac{1}{2}$ minutes
- e. It is not possible

9. If a quadratic equation has integer coefficients, which of the following can **not** be the discriminant?

- a. -23
- b. 23
- c. 24
- d. 25
- e. All of the choices can be the discriminant

10. Let $g(x)$ be a linear function such that for $f(x) = 2x^2 - 3x + 7$, the function $(f \circ g)(x) = 72x^2 - 66x + 21$. What is $g(x)$?

- a. $g(x) = 36(22x + 3)$
- b. $g(x) = 36x + 3$
- c. $g(x) = 42x - 231$
- d. $g(x) = 6x - 2$
- e. None

11. A function $f(x)$ contains the point $(-2, -5)$. Which of these functions **must** contain the point $(8, -1)$?

- a. $g(x) = f(-4x) - 4$
- b. $g(x) = f(-4x) + 4$
- c. $g(x) = f(-\frac{x}{4}) - 4$
- d. $g(x) = f(-\frac{x}{4}) + 4$
- e. None of these

12. When dividing $6x^3 + 7x^2 + 8x + 9$ by $3x + 12$, the remainder is ...

- a. 3
- b. $\frac{3}{4}$
- c. -295
- d. -9447
- e. The operation cannot be performed

13. On what interval(s) is the graph of $f(x)$ increasing? $f(x) = \begin{cases} 3x + 5 & , \text{ if } -3 < x < 2 \\ 2x^2 + 5x - 7 & , \text{ otherwise} \end{cases}$?

- a. $(-3, 2)$
- b. $(-3, \infty)$
- c. $(\infty, 2)$
- d. $(-\infty, -3) \cup (2, \infty)$
- e. $(4, 11)$

14. A grocer wants to display items so that there is one item on the top row, two items on the second row, three on the third, etc. If there are 2020 items available, what is the tallest display possible?

- a. 45 rows
- b. 63 rows
- c. 202 rows
- d. 1010 rows
- e. None of these

15. What changes need to be made to the graph of $y = x^2$ to sketch the graph of $f(x) = -3x^2 + 6x - 1$?
- vertical reflection, vertical stretch by factor of 3, shift right 6, and shift down 1
 - vertical reflection, vertical stretch by factor of $\sqrt{3}$, shift right 6, and shift down 1
 - vertical reflection, vertical stretch by factor of 3, shift right 2, and shift down 1
 - vertical reflection, vertical stretch by factor of 3, shift left 2, and shift up 1
 - vertical reflection, vertical stretch by factor of 3, shift right 1, and shift up 2
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16. For what value of x does $\log_x\left(\frac{x}{2} + 18\right) = 2$?

- a. -4 b. $\frac{9}{2}$ c. 12 d. Both a and b e. None of these
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17. A shopkeeper buys shirts wholesale for 30% below the suggested retail price. They want to mark the shirts such that, after giving a 25% discount, they will still make a 20% profit. What price should they put on the shirts?

- 75 % of the suggested retail price.
 - The suggested retail price.
 - 112% of the suggested retail price.
 - 135% of the suggested retail price.
 - None of these
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18. A shape is called **equable** if its perimeter and area are numerically equal. For example, an equable square has side length 4 giving it both a perimeter of 16 and area of 16. What is the area of an equable circle?

- a. 2 b. 4 c. 2π d. 4π e. $16\pi^2$
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19. An animal shelter has a certain number of dogs and cats. One day 18 cats are adopted, leaving twice as many dogs as cats. The next day 20 dogs are adopted, leaving 3 times as many cats as dogs. How many total animals were there to begin with?

- a. 43 b. 54 c. 94 d. 138 e. None of these
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20. What is the domain of $f(x) = \sqrt[3]{\frac{x+2}{x^2-9}}$?

- a. $(-3, -2] \cup (3, \infty)$ b. $(-\infty, -3) \cup (3, \infty)$ c. $[-2, \infty)$ d. $x \neq \pm 3$ e. None of these
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21. What is $\frac{1}{4} + \frac{3}{4^2} + \frac{1}{4^3} + \frac{3}{4^4} + \frac{1}{4^5} + \frac{3}{4^6} + \dots = ?$

- a. $\frac{7}{15}$ b. $\frac{1}{2}$ c. $\frac{\pi}{6}$ d. ∞ e. None of these
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22. For what value of p is $x = \frac{1}{\sqrt{3}}$ a root of the equation $px^2 + (\sqrt{3} - \sqrt{2})x - 1 = 0$

- a. 2 b. $\sqrt{6}$ c. $\frac{1}{\sqrt{2}-\sqrt{3}}$ d. 1 e. None of these

23. If the six-digit number, $20 \star \otimes 20$, is divisible by 99, what does $|\star - \otimes| = ?$

- a. 1 b. 2 c. 3 d. 4 e. 5

24. If $3^x = 16$ and $2^y = 9$, what is xy ?

- a. 4 b. 6 c. 8 d. 12 e. 24

25. For a set of distinct real numbers $\{a, b, c, d, e\}$ that satisfy the following equations, what are the greatest and least members of the set?

$$a - 2c + e = 0,$$

$$b - 2a + d = 0,$$

$$c - 2d + e = 0.$$

- a. $\{a, b\}$ b. $\{b, d\}$ c. $\{b, e\}$ d. $\{d, e\}$ e. None of these

26. How many solutions does $|x - 9| - |11 - x^2| = 0$ have?

- a. 0 b. 1 c. 2 d. 3 e. 4

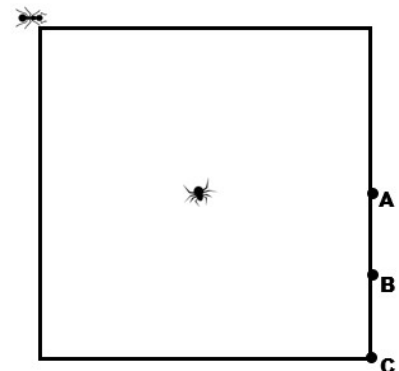
27. If d and t are, respectively, the doubling and tripling time of an exponentially increasing population, which of these is true ?

- a. $d^3 = t^2$ b. $3e^d = 2e^t$ c. $e^{3d} = e^{2t}$ d. $\log_3 d = \log_2 t$ e. $3^d = 2^t$

28. An ant in the corner of a square room challenges a spider, in the center of the room, to a race. The ant says,

“Mr Spider, I will race you to any point on the wall that you wish and since I am faster than you, I will stay on the wall while you run straight to the finish.”

If the spider runs 8 in/sec and the ant 25 in/sec, which point should the spider choose as the finish?



- a. A b. B c. C d. Any of them e. None of them