

ICTM Algebra II 2017

- On Tuesday, St. Elmo's Steakhouse sold 6 porterhouse steaks, 4 ribeye steaks, and 3 flat iron steaks for a total menu price of \$636. On Wednesday, St. Elmo's sold 5 porterhouses, 5 ribeyes, and 4 flat irons for a total of \$658. On Thursday, they sold 2 porterhouses, 7 ribeyes, and 6 flat irons for a total of \$643. Of these three cuts, how much more does the most expensive cost when compared to the least expensive?

a. \$3                      b. \$17                      c. \$18                      d. \$20                      e. \$23
- Which of the following expressions is equivalent to  $\sqrt[4]{(\sqrt[3]{x^8})^5}$ ?

a.  $x^3\sqrt[3]{x}$                       b.  $x^2\sqrt[3]{x^2}$                       c.  $x^5\sqrt{x^3}$                       d.  $x^3\sqrt[3]{x^2}$                       e.  $x^2\sqrt{x^3}$
- For a certain equation,  $x$  varies jointly with  $y$  and inversely with  $z^2$ , and if  $y = 8$  and  $z = 4$ , then  $x = 6$ . What must  $y$  be if  $x = 2$  and  $z = 6$ ?

a. 12                      b. 9                      c. 8                      d. 6                      e. 4
- What shape is formed by the equation  $4y^2 + 3x + 1 = -2y - 4x^2 + 2$ ?

a. circle                      b. hyperbola                      c. ellipse                      d. parabola                      e. No points satisfy the equation
- If  $f^{-1}(x) = \frac{3x+2}{x-5}$ , for what value(s) of  $x$  is  $f(x)$  not defined?

a.  $x = 5$                       b.  $x = -5$                       c.  $x = -\frac{2}{3}$                       d.  $x = -\frac{2}{5}$                       e.  $x = 3$
- Simplify the following rational expression:  $\frac{\frac{2}{x+1} + \frac{1}{x+2}}{\frac{x+1}{x+2} - \frac{1}{x+1}}$ .

a. 5                      b.  $\frac{3x+5}{3x}$                       c.  $\frac{3x+5}{3x+3}$                       d.  $\frac{3x+5}{x}$                       e.  $\frac{3x+5}{3}$
- If the line  $y = x + 3$  intersects the ellipse  $\frac{x^2}{9} + y^2 = 1$  at  $(a, b)$  and  $(c, d)$ , find  $a + b + c + d$ .

a. -8                      b.  $1\frac{1}{5}$                       c.  $4\frac{2}{3}$                       d. 0                      e.  $-4\frac{4}{5}$
- If  $a$  and  $b$  are solutions to  $\frac{2}{x+2} + \frac{4}{x-3} = 1$ , find  $ab$ .

a. -8                      b. -6                      c.  $\frac{5}{2}$                       d. 48                      e. This equation has fewer than two solutions

9. If  $f(x)$  is a linear function that is reflected across the line  $y = x$ , which of the following must be true?
- If  $f(a) = b$ , then  $a \neq b$  except for at most one value of  $a$ .
  - If  $f(a) = b$ , then the point  $(b, a)$  must be on the reflected graph.
  - $f(x)$  and its reflection must intersect each other at exactly one point.
  - The reflection must also be a function.
  - All of the above must be true.

10. In a particular forest, there currently is a population of 500 squirrels which is declining at an average rate of 3% annually. If we were to use an equation to model this decay, which of the following would we use?

a.  $y = 500(1.03)^x$       b.  $y = 500(.97)^x$       c.  $y = 500(.3)^x$   
 d.  $y = 500(.03)^x$       e.  $y = 500(.03)^{-x}$

11. Simplify  $10^{\log(6)} + 3 \log(2) + \log(15) - \frac{1}{2} \log(144)$ .

a.  $48 + \log\left(\frac{5}{4}\right)$       b.  $6 + \log(11)$       c.  $\log(11)^6$       d. 42      e. 7

12. If the equation  $y = 2x^2 - 8x + 9$  were rewritten in the form  $y = a(x - h)^2 + k$  for appropriate values of  $a$ ,  $h$ , and  $k$ , which of the following expressions is largest?

a.  $ahk$       b.  $k^{h^a}$       c.  $a + h + k$       d.  $a - h - k$       e.  $\frac{k}{\frac{h}{a}}$

13. To get in shape, Benjamin decided to do 7 pushups on January 1<sup>st</sup>, and to do 2 more pushups each day than the previous day for the full month of January. Which expression best represents the total number of pushups Benjamin did that month?

a.  $7 + \sum_{n=1}^{31} 2$       b.  $\sum_{n=1}^{31} 5 + 2n$       c.  $5 + \sum_{n=1}^{31} 2n$       d.  $\sum_{n=1}^{31} 7 + 2^n$       e.  $\sum_{n=1}^{31} 7 + 2n$

14. The mean of a data set of size nine is 72. If an outlier is removed, the mean of the remaining data is 80. What is the value of the outlier?

a. 2      b. 8      c. 27      d. 64      e. 72

15. If  $r_0 = 4$  and  $r_n = 3r_{n-1}$  for  $n \geq 1$ , which of the following is an equivalent way to write  $r_n$ ?

a.  $r_n = 1 + 3n$       b.  $r_n = 4 + 3n$       c.  $r_n = 12n$       d.  $r_n = 4 + 3^n$       e.  $r_n = 4 \times 3^n$

16. Prior to the potato famine, in 1841 Ireland had a population of 8 million. Just ten years later, Ireland's population had decreased to 6.5 million. Assuming the population continued this exponential decay, approximately what would the population have been in 1861 rounded to the nearest hundred-thousand?

a. 6 million      b. 5.7 million      c. 5.5 million      d. 5.3 million      e. 5 million

17. Which of the following statements are true for all real values of  $x$ ?

I.  $-\sqrt{|x|} \leq 0$       II.  $\ln(x^2 + 1) \geq 0$       III.  $\pi^{-x} > 0$

- a. I, II, and III
- b. I and II only
- c. I and III only
- d. I only
- e. III only

18. If  $a$  and  $b$  are chosen so that  $a < b$  and the piecewise function given below is continuous, find  $a + b$ .

$$f(x) = \begin{cases} \sqrt{4-x} & \text{if } x \leq a \\ \left| \frac{x}{4} - 2 \right| & \text{if } a < x < b \\ (x-12)^2 + 1 & \text{if } x \geq b \end{cases}$$

- a.  $-12$       b.  $0$       c.  $8$       d.  $12$       e.  $243$

19. For some right triangle, the length of the longer leg is one meter less than three times the length of the shorter leg, and the length of the hypotenuse is two meters more than the length of the longer leg. What is the area of the triangle?

- a.  $666 \text{ m}^2$       b.  $444 \text{ m}^2$       c.  $420 \text{ m}^2$       d.  $222 \text{ m}^2$       e.  $210 \text{ m}^2$

20. A jar contains 8 marbles, 6 red and 2 black. A second jar contains 5 red marbles and an unknown number of black marbles. If a single marble is drawn from each jar, the probability that both are the red is  $\frac{1}{4}$ . How many black marbles are in the second jar?

- a. 5      b. 7      c. 8      d. 9      e. 10

21. Which of the following functions has no  $x$ -intercept, has a  $y$ -intercept of 3, tends to infinity as  $x$  tends to negative infinity, and tends to zero as  $x$  tends to infinity?

a.  $y = -\frac{x^3}{2} + 3$       b.  $y = \frac{x^3}{2} + 3$       c.  $y = 2x^2 + 1$       d.  $y = 3\left(\frac{1}{2}\right)^{-x}$       e.  $y = 3\left(\frac{1}{2}\right)^x$

22. Solve for  $x$  if  $|x + 5| < 1$ .

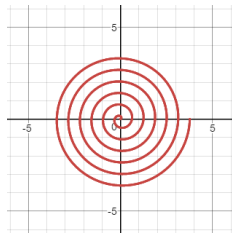
- a.  $x < 4$  or  $x > 6$
- b.  $4 < x < 6$
- c.  $-6 < x < -4$
- d.  $x < -6$  or  $x > -4$
- e.  $-6 < x < 6$

23. Based on customer feedback, The South Bend Chocolate Company knows that the number of bags of chocolate covered salty pretzels that they can sell varies linearly with the price and is given by the equation  $U(p) = 500 - 25p$ . If the maximum gross profit occurs at the midway point between the zeros of the gross profit function, for what value of  $p$  would they make the most gross profit?
- a. \$12                      b. \$7.50                      c. \$8                      d. \$20                      e. \$10
24. If  $f(x) = 2x^2 - 9x + c$  has exactly one real solution, use the discriminant to find the value of  $c$ .
- a.  $c = \frac{-9}{8}$                       b.  $c = -\frac{5}{2}$                       c.  $c = \frac{81}{8}$                       d.  $c = -\frac{81}{8}$                       e. No such  $c$  exists.
25. Find the equation of a parabola that passes through the points (1,0), (4,3), and (6, 15).
- a.  $y = 3x^2 - 14x + 11$   
b.  $y = \frac{1}{3}x^2 + \frac{2}{3}x - 1$   
c.  $y = x^2 - 3x + 1$   
d.  $y = x^2 - 4x + 3$   
e.  $y = x^2 - 2x + 1$
26. At T-Rex Pizza, pizza can be topped with pepperoni, sausage, and Canadian bacon. Every pizza they sell has either exactly one of these toppings or all three of these toppings. Two-thirds of their pizzas are served with pepperoni, two-thirds are served with sausage, and two-thirds are served with Canadian bacon. If you pick a pizza at random, what is the probability it has all three toppings?
- a.  $\frac{1}{6}$                       b.  $\frac{1}{4}$                       c.  $\frac{1}{3}$                       d.  $\frac{1}{2}$                       e.  $\frac{2}{3}$
27. If  $f(x) = 3x$ ,  $g(x) = 5^x$ , and  $h(x) = x - 2$ , then the function  $5^{3x-6}$  could be described as:
- a.  $f(g(h(x)))$                       b.  $f(h(g(x)))$                       c.  $g(f(h(x)))$                       d.  $g(h(f(x)))$                       e.  $h(f(g(x)))$
28. If  $a$  and  $b$  are two unique solutions of  $9^x \times 3^{x^2} = 27$ , find  $a \times b$ .
- a. 3                      b. -3                      c. 0                      d. -12                      e. 12
29. If  $f(x) = 2(3)^{5x}$ , what is  $f^{-1}(x)$ ?
- a.  $\frac{1}{5} \log_3 \left( \frac{x}{2} \right)$                       b.  $\frac{1}{2} \log_3 \left( \frac{x}{5} \right)$                       c.  $2 \log_3 \left( \frac{x}{5} \right)$                       d.  $\frac{1}{2} \log_3 (5x)$                       e.  $2 \log_3 (5x)$
30. If  $a, b$  are real numbers, and  $(4 + 2i)(a + bi)$  is a real number, what is  $\frac{a}{b}$ ?
- a. -2                      b. 2                      c.  $\frac{3}{2}$                       d.  $-\frac{1}{2}$                       e.  $\frac{1}{2}$

31. What is the remainder when  $x^3 + 4x - 7$  is divided by  $x + 2$ ?
- a.  $-7$                       b.  $-11$                       c.  $-3$                       d.  $42$                       e.  $-23$
32. In how many different ways can 5 cards be drawn from a standard deck of 52 playing cards if the order the cards are drawn in does not matter?
- a. 260                      b. 270,725                      c. 2,598,960                      d. 20,358,520                      e. 311,875,200
33. What transformations must be done to  $f(x) = 3^x$  to turn it into  $3^{2x} - 4$ ?
- a. Shrink the graph of  $f(x)$  vertically by a factor of 2 and shift it left 4.  
b. Stretch the graph of  $f(x)$  horizontally by a factor of 2 and shift it down 4.  
c. Shrink the graph of  $f(x)$  horizontally by a factor of 2 and shift it down 4.  
d. Stretch the graph of  $f(x)$  horizontally by a factor of 2 and shift it right 4.  
e. None of the above.
34.  $(x + y - z)(z - y - x) =$
- a.  $x^2 - y^2 + z^2 - 2xz$   
b.  $x^2 + y^2 + z^2 - 2xy - 2xz + 2yz$   
c.  $-x^2 - y^2 - z^2 - 2xy + 2xz + 2yz$   
d.  $-x^2 + y^2 - z^2 + 2xz$   
e.  $-x^2 - y^2 - z^2 + 2xy - 2xz + 2yz$

35. Which of the following represent a function?

I.



II.

Animal	Legs
Ant	6
Dog	4
Octopus	8
Cat	4

III.

$$y = -\sqrt{1 - x^2}$$

- a. II only  
b. III only  
c. I and III only  
d. II and III only  
e. None of these are functions

36. Simplify the following expression:  $\left(\frac{x^2y^{-3}}{x^5y^7}\right)^2$ .

- a.  $\frac{1}{x^6y^{20}}$                       b.  $x^5y^6$                       c.  $\frac{1}{x^3y^{12}}$                       d.  $\frac{x^{14}}{y^8}$                       e.  $\frac{1}{x^5y^8}$