



Algebra II / Integrated Math III

2012

Sponsored by the Indiana Council of Teachers of Mathematics

Indiana State Mathematics Contest

This test was prepared by faculty at **Indiana State University**

ICTM Website

<http://www.indianamath.org/>

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Next year's math contest date: April 27, 2013

1. Solve for x : $\frac{x-2}{x+3} \geq 0$

A) $-3 \leq x < 2$

B) $x < -3$ or $x \geq 2$

C) $-2 \leq x < 3$

D) $x \leq -3$ or $x \geq 2$

E) none of these

2. Find $P(1+i)$ given that $P(x) = 2x^3 - x^2 + 3x + 1$.

A) $2 + 9i$

B) 0

C) $1 + i$

D) $-1 + i$

E) $5i$

3. $(x - y + z)^2 =$

A) $x^2 + y^2 + z^2 - 2xy - 2xz + 2yz$

B) $x^2 - y^2 - z^2 - 2xy + 2xz - 2yz$

C) $x^2 + y^2 + z^2 - 2xy + 2xz - 2yz$

D) $x^2 - y^2 + z^2$

E) none of these

4. Evaluate: $\sqrt{-36} \left(\sqrt{-81} - \frac{\sqrt{8}}{\sqrt{-18}} \right)$.

A) -50

B) 58

C) $58i$

D) -58

E) $-50i$

5. Write the following without radicals in the denominator: $\frac{\sqrt{7}}{\sqrt{7}+2}$

A) $\frac{7-2\sqrt{7}}{3}$

B) $\frac{7+2\sqrt{7}}{3}$

C) $\frac{2}{7}$

D) $\frac{\sqrt{7}-2}{9}$

E) none of these

6. What is the reciprocal of $\sqrt{2} + \sqrt{3} - \sqrt{5}$?

A) $\sqrt{2} + \sqrt{3} + \sqrt{5}$

B) $\frac{2\sqrt{3} + 3\sqrt{2} + \sqrt{30}}{12}$

C) $\frac{15\sqrt{2} + 10\sqrt{3} - 6\sqrt{5}}{30}$

D) $\sqrt{5} - \sqrt{3} - \sqrt{2}$

E) $\frac{3\sqrt{2} + 2\sqrt{3} + \sqrt{30}}{6}$

7. Find a quadratic function $f(x) = ax^2 + bx + c$ having minimum value -9 and zeros $\frac{1}{2}$ and $-\frac{5}{2}$. The coefficients a, b, c are relatively prime integers. Find the sum $a + b + c$.
- A) 17 B) -9 C) 7 D) 1 E) none of these
8. Which inequality is false for every real number?
- A) $r^2 - 4r \geq 5$ B) $(3z - 1)(z - 2) \leq 6$ C) $t^3 + 9t \leq 0$
- D) $s^3 < 1$ E) $9u^2 + 49 < 42u$
9. Solve $27^{2t-1} = 81^{t+2}$.
- A) 3 B) -3 C) $-\frac{1}{2}$ D) -2 E) $\frac{11}{2}$
10. Which expression below is equivalent to $-3\log_8 4$?
- A) $\frac{\sqrt[5]{32^4}}{32}$ B) $\ln \frac{1}{e^2}$ C) $\sqrt{-4}$
- D) $\log 1 - \log 0.01$ E) $\log_5 \sqrt{5} - \log_5 5\sqrt{5}$
11. An ellipse with equation $\frac{x^2}{16} + \frac{y^2}{36} = 1$ has which point as a focus?
- A) $(0, -2\sqrt{3})$ B) $(0, 2\sqrt{5})$ C) $(-2\sqrt{5}, 0)$ D) $(2\sqrt{13}, 0)$ E) $(0, 6)$
12. How many real number solutions are there to the equation $(x^2 - 4x + 2)^{x^2-1} = 1$?
- A) 2 B) 3 C) 4 D) 5 E) 6

13. The domain of the function $f(x) = \frac{x+1}{x} - \frac{3}{\sqrt{5-x}}$ is
- A) $x \neq 0$ B) $-\infty < x < \infty$ C) $x > 5$ and $x \neq 0$
- D) $x < 5$ and $x \neq 0$ E) none of these
14. If $f(x) = \frac{4}{x} - 1$, find $f^{-1}(7)$ if possible.
- A) 0.5 B) 7 C) 4 D) 0 E) does not exist
15. Solve for u in terms of v if $v = \frac{2u-1}{u-5}$.
- A) $u = \frac{2v-1}{v-5}$ B) $u = \frac{\frac{1}{2}v+1}{v+5}$ C) $u = \frac{v-5}{2v-1}$ D) $u = \frac{5v-1}{v-2}$ E) $u = \frac{5v+1}{v+2}$
16. If 40 mg of a radioactive substance decays to 5 mg in 12 minutes, find the half-life, in minutes, of the substance.
- A) 2 B) 3 C) 4 D) 6 E) 8
17. Solve $e^{2x-5} = 71$.
- A) $\frac{\ln 71-5}{2}$ B) $\frac{\ln 71}{2x-5}$ C) $\ln 71+5$ D) $\frac{\ln 71+5}{2}$ E) none of these
18. Let $\log_7(a \cdot b) + \log_7(b \cdot c) + \log_7(a \cdot c) = 10$ for positive a , b , and c . What is the value of $a \cdot b \cdot c$?
- A) 49 B) 7 C) 16807 D) $\sqrt{343}$ E) none of these
19. To measure the speed of the jet stream, a weather plane left its base at noon and flew 800 km directly against the stream with an airplane speed of 750 km/h. It then returned directly to its base, arriving at 2:24 p.m. What was the speed of the jet stream?
- A) 100 km/h B) 150 km/h C) 200 km/h D) 250 km/h E) 300 km/h

20. If $\log_b z = \frac{1}{3} \log_b x + \log_b y$, write z in terms of x and y .
- A) $y\sqrt[3]{x}$ B) $(x+y)^{\frac{1}{3}}$ C) $(xy)^{\frac{1}{3}}$ D) $\frac{x}{3} + y$ E) none of these
21. Suppose z varies directly as x and inversely as the square of y . If $z = 2$ when $x = 36$ and $y = 3$, find z when $x = 24$ and $y = 2$.
- A) 12 B) 6 C) 3 D) 2 E) none of these
22. What is the sum of all the coefficients of the terms of the expansion of $(3x - 4y)^9$?
- A) 1 B) -1 C) 144 D) -1728 E) none of these
23. Find the sum of the infinite geometric series $64 + 48 + 36 + 27 + \dots$ if it has one.
- A) $\frac{256}{7}$ B) $\frac{256}{3}$ C) 256 D) no sum E) none of these
24. Find the roots of $y^{-2} - 2y^{-1} + 2 = 0$
- A) $\frac{1 \pm i}{2}$ B) $1 \pm i$ C) $-1 \pm i$ D) $\frac{-1 \pm i}{2}$ E) none of these
25. Find the number of real solutions of this system:
$$\begin{cases} x^2 + 16y^2 = 25 \\ xy - 3 = 0 \end{cases}$$
- A) 0 B) 1 C) 2 D) 3 E) 4
26. At 1:00 p.m. Sue left her home and began walking at 6 km/h toward Sandy's house. Fifteen minutes later, Sandy left her home and walked at 8 km/h toward Sue's house. If they live 5 km apart, at what time did they meet?
- A) 1:30 p.m. B) 1:35 p.m. C) 1:40 p.m. D) 1:45 p.m. E) none of these

27. Solve $A = \frac{h}{2}(b_1 + b_2)$ for b_2 .

- A) $\frac{2A}{h} - b_1$ B) $\frac{A}{2h} - b_1$ C) $A - \frac{hb_1}{2}$ D) $\frac{2A - b_1}{h}$ E) $\frac{2A}{hb_1}$

28. Three of the vertices of square $ABCD$ have coordinates $A(-3, -1)$, $B(-1, 4)$ and $C(4, 2)$. The product of the x - and y -coordinate of the fourth vertex is

- A) -6 B) -4 C) 4 D) 6 E) none of these

29. Suppose $f(x) = e^{2x} + 1$ and $f(g(x)) = g(f(x)) = x$ for appropriate values of x . Then $g(x) =$

- A) $2 \cdot \ln x + 1$ B) $\frac{1}{2} \cdot \ln(x + 1)$ C) $2 \cdot \ln(x - 1)$ D) $\ln(x - 1)$ E) none of these

30. What is the distance from $(1, 1)$ to the midpoint of the segment from $(3, 4)$ to $(5, 6)$?

- A) $\frac{\sqrt{41} - \sqrt{13}}{2}$ B) $\sqrt{2}$ C) 5 D) $\frac{\sqrt{41} + \sqrt{13}}{2}$ E) none of these

31. Suppose that $s = \frac{10t}{t+6}$. For what value(s) of t is $s > t$?

- A) $t > -4$ B) $t < -6$ C) $0 < t < 4$ D) $t > 4$ E) $0 < t < 4$ or $t < -6$

32. If x and y are real numbers with $x \geq y$, which of the following must be true?

I. $|x - y| \geq x - y$

II. $x + y \geq 0$

III. $\frac{x}{y} \geq 1$

- A) I only B) I and II C) III only
D) I and III E) I, II, and III

33. If $f(x) = \frac{1}{x-1}$, then $f(f(x)) =$

- A) $x-1$ B) $\frac{2}{x-1}$ C) $\frac{1}{2(x-1)}$ D) $\frac{1}{x^2-2x+1}$ E) $\frac{x-1}{2-x}$

34. The graph of $(x+a)^2 = 4(y+2a)$ intersects the x -axis at $x=r$ and $x=s$. Find $s+r$.

- A) $-\sqrt{8a}$ B) $-\sqrt{8a}$ C) $-2a$ D) $-\sqrt{2a}$ E) $-\sqrt{8a}$

35. Solve for x : $3^{2x} + 3^x - 20 = 0$.

- A) $\frac{\ln 3}{\ln 4}$ B) $\frac{\ln 3}{\ln 20}$ C) $\frac{\ln 20}{\ln 3}$ D) $\frac{\ln 4}{\ln 3}$ E) none of these

36. Cassandra set her watch to the correct time at 3 p.m. At the actual time of 4:00 p.m., she noticed that her watch read 3:57:36 p.m. Assuming that her watch loses time at a constant rate, what will be the actual time when her watch first reads 1 a.m.?

- A) 1:22:24 a.m. B) 1:24 a.m. C) 1:25 a.m. D) 1:27 a.m. E) 1:30 a.m.