Algebra I / Integrated Math I

2010

Sponsored by the Indiana Council of Teachers of Mathematics

Indiana State Mathematics Contest

This test was prepared by faculty at **University of Southern Indiana**

ICTM Website
http://www.indianamath.org/

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Next year's math contest date: April 23, 2011

1)   Solve the equation: $x - 3(x+3) = 5(2x+3)$

     A) $\{-2\}$ B) $\{-1\}$ C) $\left\{-\dfrac{1}{2}\right\}$ D) $\left\{\dfrac{1}{2}\right\}$ E) $\{1\}$

2)   Find the product: $(x-2)(x+2)^2$

     A) $x^3 - 8$
     B) $x^3 + 8$
     C) $x^3 - 2x^2 + 4x - 8$
     D) $x^3 + 2x^2 - 4x - 8$
     E) $x^3 + 2x^2 + 4x - 8$

3)   Simplify: $10\left[\left(3^{-1} + 2^{-1}\right)^{-1}\right]$

     A) -50 B) -12 C) -2 D) 12 E) 50

4)   Sixty miles per hour is equal to: (Note: 5280 feet = 1 mile)

     A)   88 feet per minute
     B)   88 feet per second
     C)   3600 feet per minute
     D)   3600 feet per second
     E)   5280 feet per second

5)   Solve the equation: $\dfrac{2x-1}{6x-7} = \dfrac{x+1}{3x-4}$

     A) $\left\{-\dfrac{11}{3}\right\}$ B) $\left\{-\dfrac{11}{10}\right\}$ C) $\left\{\dfrac{11}{10}\right\}$ D) $\left\{\dfrac{11}{3}\right\}$ E)   none of these

6)   Solve the equation: $\sqrt{4x-3} - 1 = 2$

     A) $\{0\}$ B) $\{2\}$ C) $\{3\}$ D) $\varnothing$ E)   none of these

7)   Find the values for a such that $ax^2 + 2x + 4 = 0$ has no real solutions.

    A) $\left(-\infty, -\dfrac{1}{4}\right)$ B) $\left(-\infty, \dfrac{1}{4}\right)$ C) $\left(-\dfrac{1}{4}, \infty\right)$ D) $\left(\dfrac{1}{4}, \infty\right)$ E) none of these

8)   Solve the equation: $2(x-1)^2 = 14$

    A) $\left\{-1 \pm \sqrt{7}\right\}$ B) $\left\{1 \pm \sqrt{7}\right\}$ C) $\left\{-1 \pm 2\sqrt{3}\right\}$ D) $\varnothing$ E) none of these

9)   The expression $(y-1)^4 - (y-1)^2$ is equivalent to which of the following:

    A) $y(y-1)^2(y-2)$

    B) $y(y^2 - 2y + 1)(y+2)$

    C) $(y^2 - 2)(y^2 + 1)$

    D) $y^4 - 4y^3 + 5y^2 - 2y + 1$

    E) $y(y^2 - 3y + 2)(y+1)$

10)   The expression $\left(a^{4x} - b^{3y}\right)^2$ is equivalent to which of the following:

    A) $a^{16x^2} + b^{9y^2}$

    B) $a^{8x} + b^{6y}$

    C) $a^{8x} - a^{4x}b^{3y} + b^{6y}$

    D) $a^{8x} - 2a^{4x}b^{3y} - b^{6y}$

    E) $a^{8x} - 2a^{4x}b^{3y} + b^{6y}$

11)   Solve the system of equations, $\begin{cases} 2x + 4y = 25 - x \\ 3x - 2y = 19 \end{cases}$, for x and y.

    A) $\{(3, -5)\}$ B) $\{(5, 5)\}$ C) $\left\{\left(\dfrac{43}{7}, \dfrac{33}{7}\right)\right\}$ D) $\{(7, 1)\}$ E) $\{(9, 4)\}$

12)   Solve the equation: $\dfrac{9t+6}{t(t+3)} = \dfrac{7}{t+3}$

    A) $\{-3\}$ B) $\{0\}$ C) $\{2\}$ D) $\{3\}$ E) $\varnothing$

13)   Based on the graph of $f(x)$ which of the following statements is FALSE?
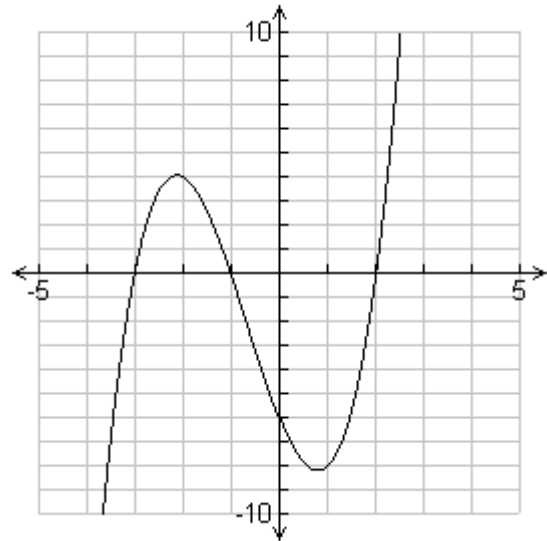
    A) -3 is a zero of $f(x)$.

    B) 2 is a solution of $f(x)=0$.

    C) $x+3$ is a factor of $f(x)$.

    D) $(-1,0)$ is an x-intercept of the graph of $f(x)$.

    E)   None of these are false.

14)   The point $(-1,-1)$ is on the line of which of the following equation(s)?

    A) $y=-x$

    B) $2y+2=0$

C) $2y = -x$

D)   Both A and C

E)   Both B and C

15)   If a line with equation $3x - ky = 4$ has a slope of $\dfrac{5}{6}$, find the value of k.

A) -4 B) $-\dfrac{4}{3}$ C) $\dfrac{5}{6}$ D) 3 E) $\dfrac{18}{5}$

16) Which equation below represents the line that passes through the point $(-2,0)$ and is also perpendicular to the line $2x+5y=10$?

    A) $x+y=-2$

    B) $x-2y=4$

    C) $2x-5y=0$

    D) $5x+2y=4$

    E) $5x-2y=-10$

17) A local computer outlet sells 3 Kompack and 2 Dull computers for $1625, or 2 Kompack and 5 Dull computers for $2000. What is the price of a single Kompack and a single Dull computer?

    A) $400 per Kompack and $365 per Dull

    B) $375 per Kompack and $250 per Dull

    C) $350 per Kompack and $375 per Dull

    D) $325 per Kompack and $286 per Dull

    E) $250 per Kompack and $350 per Dull

18) Given the following system of inequalities, what is the shape of the solution set?

$$\begin{cases} x \le 4 \\ y \le 6 \\ x-y \ge -2 \\ 2x+5y \ge 10 \end{cases}$$
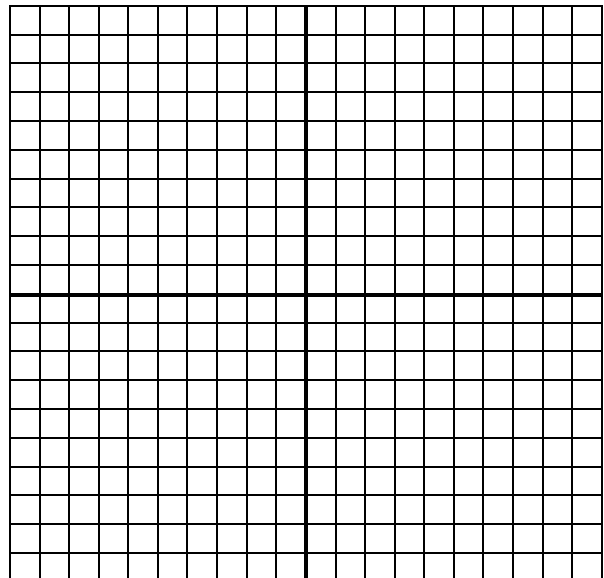
    A)   a triangle

    B)   a square

    C)   a rectangle

    D)   a trapezoid

    E)   a pentagon

19)   Solve the inequality: $7|5-4x| \geq 91$

   A) $(-\infty, -2]$

   B) $(-\infty, -2] \cup \left[\dfrac{9}{2}, \infty\right)$

   C) $(-\infty, -2] \cap \left[\dfrac{9}{2}, \infty\right)$

   D) $\left[-2, \dfrac{9}{2}\right]$

   E) none of these

20)   Solve the inequality: $-2 \leq 7 - \dfrac{3}{4}x < 9$

   A) $\left(-\dfrac{8}{3}, 12\right]$

   B) $\left(-\dfrac{29}{3}, 5\right]$

   C) $\left(-\infty, -\dfrac{29}{3}\right) \cup [5, \infty)$

   D) $\left(-\infty, -\dfrac{8}{3}\right) \cup [12, \infty)$

   E) none of these

21)   Solve the equation: $\sqrt{x - \sqrt{2x^2 - 7}} = 1$

   A) $\{2\}$ B) $\{-4\}$ C) $\{-4, 2\}$ D) $\left\{-\dfrac{2}{3}, 2\right\}$ E) none of these

22)   Solve the given equation and determine the sum of the two solutions: $4x^2 - 25x - 56 = 0$

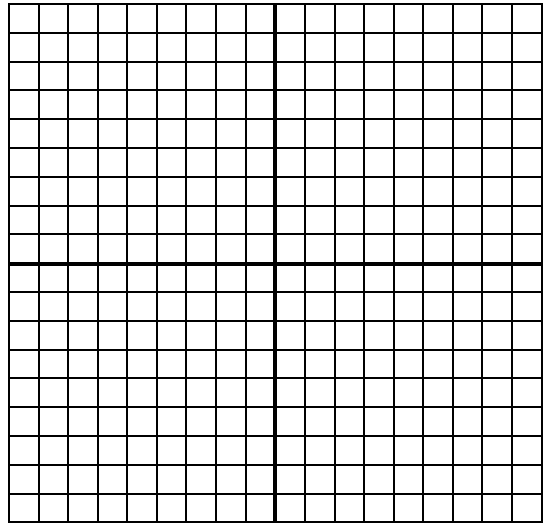   A) $-\dfrac{25}{4}$ B) $-\dfrac{5}{2}$ C) $\dfrac{5}{2}$ D) $\dfrac{1}{2}$ E) $\dfrac{25}{4}$

23) Line l contains the point, $(1,-4)$, and is perpendicular to the line, $x+2y=6$. In which quadrants does the graph of line l lie?

A) I, II and III

B) I, II, and IV

C) I , III, and IV

D) I and III

E) I, II, III, and IV

24) Which equation below represents the line that passes through the midpoint of the segment connecting the points $(-1,-5)$ and $(13,-1)$ and is also parallel to the line $3x-5y=10$?

A) $3x-5y=33$

B) $3x+5y=33$

C) $5x-3y=39$

D) $5x+3y=39$

E) none of these

25) The graphs of $3x+2y=6$ and $2x+3y=6$ intersect at which of the following points?

A) $(-2,6)$ B) $\left(\dfrac{5}{6},\dfrac{5}{6}\right)$ C) $(1,1)$ D) $\left(\dfrac{6}{5},\dfrac{6}{5}\right)$ E) $(2,0)$

26) If $f(x)=-x^2+5x+2$, find $3f\left(-\dfrac{1}{3}\right)$

A) -4 B) $-\dfrac{2}{3}$ C) $\dfrac{2}{9}$ D) $\dfrac{2}{3}$ E) $\dfrac{4}{3}$

27) Simplify: $7xy + 2x^2 + 5y - 1 - 3(x^2 + 2y - 4) - x(3x - 4y + 2)$

    A) $3xy - 4x^2 + 2x + 11y - 13$

    B) $3xy - 4x^2 + 2x + 11y + 13$

    C) $7xy - 4x^2 + 3y - 3$

    D) $11xy - 4x^2 - 2x + 11$

    E) $11xy - 4x^2 - 2x - y + 11$

28) In the box on the right, which ones represent functions?

    A) II and IV
    B) III and IV
    C) I, II and III
    D) I, III and IV
    E) I and III

I. $\{(-5,7),(-2,-1),(3,-6),(4,7)\}$

II. $x = y^2 + 2y + 1$

III. $y = x^2 + 2x + 1$

IV.

Sam Worthington → Avatar
Channing Tatum → Poor Things
→ Dear John
Paul Bettany → Legion
John Travolta → From Paris with Love

29) Determine the domain of the function: $f(x) = \dfrac{x-5}{\sqrt{x+2}}$

    A) $\{x \mid x \text{ is a real number}, x \neq -2, x \neq 5\}$

    B) $\{x \mid x \text{ is a real number}, x > -2\}$

    C) $\{x \mid x \text{ is a real number}, x \neq 5, x > -2\}$

    D) $\{x \mid x \text{ is a real number}, x \geq -2\}$

    E) $\{x \mid x \text{ is a real number}\}$

30)   Factor : $x^2 - 5x - 6$

     A) $(x-6)(x+1)$
     B) $(x-3)(x-2)$
     C) $(x-2)(x+3)$
     D) $(x-1)(x+6)$
     E) $(x+3)(x+2)$

31)   Solve the equation: $(x+6)^2 = 48$

     A) $\left\{-2\sqrt{2}, 2\sqrt{2}\right\}$
     B) $\left\{-6-4\sqrt{3},\ -6+4\sqrt{3}\right\}$
     C) $\left\{-6-2\sqrt{6},\ -6+2\sqrt{6}\right\}$
     D) $\{-3,\ 4\}$
     E) $\varnothing$

32)   Simplify: $\dfrac{6p+9q}{3} - \dfrac{7p+21q}{7}$

     A) 0
     B) p
     C) $p+6q$
     D) $21p$
     E) $21p+126q$

33)   Find $P(x)-Q(x)$, given that $P(x)=(x+1)^3$ and $Q(x)=(x-1)^2$.

     A) $x^3 - x^2$
     B) $x^3 - x^2 + 2$
     C) $x^3 - x^2 + x + 2$
     D) $x^3 + 2x^2 + 5x$
     E) $-x^3 - 2x^2 - 5x$

34)   The expression $\sqrt[3]{64x^{12}} + \sqrt[3]{27x^{27}}$ is equivalent to which of the following:

     A) $\sqrt[3]{91x^{39}}$

     B) $\sqrt[6]{91x^{39}}$

     C) $4x^4 + 3x^9$

     D) $4x^4 + 3x^3$

     E) $7x^7$

35)   Blue Box Bottlers has determined they can start with a rectangular piece of cardboard that is 3 inches longer than it is wide, fold up 2 inches on all four sides of the rectangle, and make some strategic cuts near corners to allow the corners to be glued. This process will create rectangular cardboard trays suitable for packaging different drinks that the company bottles. If x is the original width of the cardboard rectangle before the sides are folded (the shorter side), determine the expression which represents the surface area of the exterior of the cardboard tray—which, true to their name, would be painted blue.

     A) $x^2 - x - 2$

     B) $x^2 - 3x - 16$

     C) $x^2 + 3x - 16$

     D) $2x^2 - 10x + 8$

     E) $2x^2 - 22x + 56$

For the remainder of the test, assume all denominators to be non-zero.

36)   Divide: $\dfrac{10a^2b - 15a^3b}{10a^2b}$

     A) $-\dfrac{1}{2}a$

     B) $-\dfrac{3}{2}a$

     C) $-\dfrac{1}{2ab}$

     D) $1 - \dfrac{3}{2}a$

     E)   none of these

37)   Solve for t: $At - B = Ct$

    A) $\dfrac{B}{A-C}$

    B) $\dfrac{B+C}{A}$

    C) $\dfrac{B}{C-A}$

    D) $\dfrac{B-C}{A}$

    E) $\dfrac{A+B}{C}$

38)   The expression $\dfrac{1}{x+y} + \dfrac{1}{x-y} - \dfrac{1}{x}$ is equivalent to which of the following:

    A) $\dfrac{x^2 + y^2}{x^3 - xy^2}$

    B) $\dfrac{x^2 - y^2}{x^3 - xy^2}$

    C) $\dfrac{1}{x}$

    D) $\dfrac{1}{3x}$

    E) $\dfrac{1}{x^3 - xy^2}$

39)   Multiply and divide as indicated, then write the rational expression in lowest terms:

$$\frac{9x^2 - 25}{x^2 - 11x + 30} \times \frac{x^2 - 4x - 12}{3x^2 + x - 10} \div \frac{3x^2 + 8x + 5}{5 - x}$$

    A) $-(x+1)$

    B) $x+1$

    C) $-\dfrac{1}{x+1}$

    D) $\dfrac{1}{x+1}$

    E) $\dfrac{5-x}{x+1}$

40)   Write the rational expression in lowest terms: $\dfrac{m^3 - mn^2}{mn^2 + m^2n - 2m^3}$

A) $\dfrac{1}{2}$

B) $\dfrac{-1}{m^2n - 2}$

C) $-\dfrac{m+n}{2m+n}$

D) $-\dfrac{1}{mn - 2}$

E) $\dfrac{m+n}{2m+n}$